Reflection and Refraction of Plane Electromagnetic Waves

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Introduction:

When an electromagnetic wave falls on the interface between two media having different values of ϵ and μ, a part of the energy is sent back into the first medium but in an altered direction of propagation (the reflected wave) and a part penetrates into the second medium (refracted wave). In reality, matters are more complicated there will be absorption, i.e., derangement of the energy to heat and irregular scattering. However, right now we shall not take these into consideration. we can study the reflection and the refraction of EM waves in some detail, specifically:

- **The kinematic** relations between the directions of the incident, the refracted, and the reflected waves.
- **The dynamical** relations between the intensities, the phases, and the polarizations of all the waves.

The kinematic properties follow immediately from the wave nature of the phenomena and from the fact that there are boundary conditions to be satisfied. But they don't depend on the detailed nature of the waves or the boundary conditions. On the other hand, the dynamic properties depend entirely on the specific nature of electromagnetic fields and their boundary conditions.

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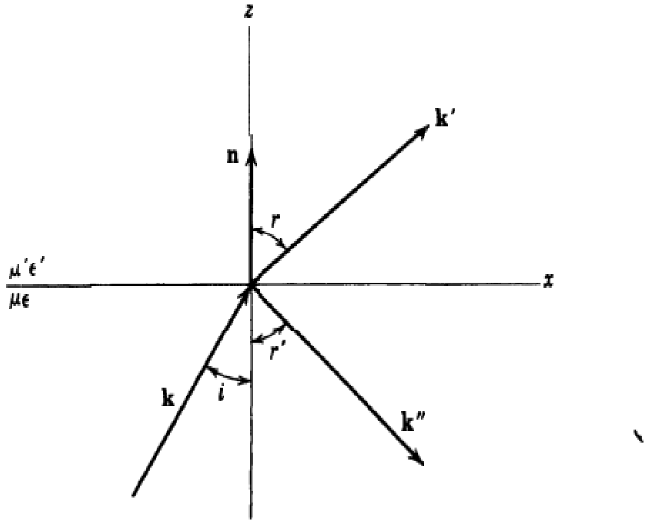
We shall take for simplicity the case of a plane wave (a parallel beam in the language of geometrical optics) and the interface also will be taken to be a plane surface labelled z = 0, so that the region z > 0 constitutes medium 1 and z < 0 constitutes medium 2 as shown in the fig.

consider plane waves of the form:

Incidence

$$E = E_0 e^{(K.x - i\omega t)} \ldots \ldots \ldots (1)$$



Incident wave **k** strikes plane interface between different media, giving rise to a reflected wave **k**″ and a refracted wave **k**′.

Refracted

$$E' = E_0{}'e^{(K'.x-i\omega t)} \dots\dots\dots(2)$$

Reflected

$$E'' = E_0{}''e^{(K''.x-i\omega t)} \dots\dots\dots(3)$$

The wave numbers have the magnitudes

$$|K| = |K''| = K = \omega\sqrt{\mu\epsilon} \dots\dots\dots(4)$$

$$|K'| = K' = \omega\sqrt{\mu'\epsilon'} \dots\dots\dots(5)$$

Where :

$$v = \frac{\omega}{K} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}\,, \quad n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

As the boundary conditions (right now we are not considering their exact form) are to be satisfied all over the interface and at all times, the three wave disturbances must depend on x, y and t in the same manner at z = 0, i.e.

$$(K.x)_{Z=0} = (K'.x)_{Z=0} = (K''.x)_{Z=0} \dots\dots\dots (6)$$

$$\omega = \omega' = \omega'' \dots\dots\dots (7)$$

for any vector **x** normal to **n**. From eq (7), we get that the frequencies of the three waves are identical. (This is not a trivial result, since the velocities of the waves being different in the two media, either the frequency or the wavelength or both must differ in the incident and refracted waves the identity of frequencies settles this question.) From eq (8), we get first that (K, K', K'') and n lie in the same plane, and secondly:

$$K \sin i = K' \sin r = K'' \sin r' \dots\dots\dots (8)$$

where i , r' and r are the angles of incidence, reflection and refraction, as indicated in the figure.

Since $K'' = K$, we find $i = r'$; the angel of incidence equals the angel of reflection. Snell's law is :

$$\frac{\sin i}{\sin r} = \frac{K'}{K} = \frac{\sqrt{\mu'\epsilon'}}{\sqrt{\mu\epsilon}} = \frac{n'}{n} \dots\dots\dots (9)$$

Now we turn our attention to the dynamical issues of intensities and phases of the reflected and the refracted waves relative to the incident wave. These issues depend on the specific nature of the wave, so let's focus on the plane electromagnetic waves:

$$E_j(x,t) = \vec{\varepsilon}_j \exp\left(iK_j.x - i\omega t\right) \dots\dots\dots (10)$$

For simplicity, let's assume that the media at both sides of the boundary are non-magnetic, $\mu_1 = \mu_2 = 1$, and have real refraction indices $n_1 = \sqrt{\epsilon_1(\omega)}$ and $n_2 = \sqrt{\epsilon_2(\omega)}$ at the frequency of the wave. Consequently, for both media

$$Z_j = \frac{Z_0 \approx 377\,\Omega}{\sqrt{\epsilon_j}} = \frac{Z_0}{n_j} \quad \dots\dots\dots (11)$$

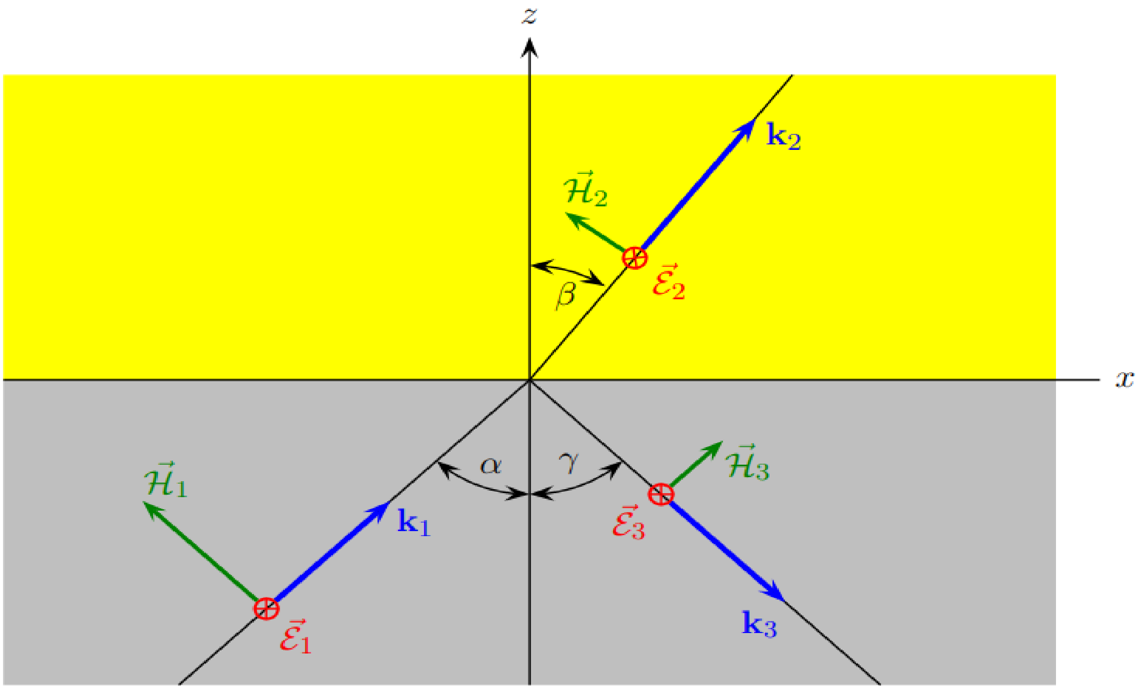
$$Z_0\vec{H}_1 = n_1\widehat{K}_1 \times \vec{\varepsilon}_1, \qquad Z_0\vec{H}_2 = n_2\widehat{K}_2 \times \vec{\varepsilon}_2, \quad Z_0\vec{H}_3 = n_3\widehat{K}_3 \times \vec{\varepsilon}_3 \dots\dots\dots (12)$$

In terms of the incident and the reflected waves at one side and the 6 refracted wave on the other side, this means

$$\vec{H}_1 + \vec{H}_3 = \vec{H}_2 \quad \dots\dots\dots (13)$$

In terms of the three waves and their amplitudes, this means:

$$\varepsilon_{1,X} + \varepsilon_{3,X} = \varepsilon_{2,X}\,, \qquad \varepsilon_{1,Y} + \varepsilon_{3,Y} = \varepsilon_{2,Y} \quad , \; {n_1}^2(\varepsilon_{1,Z} + \varepsilon_{3,Z}) = {n_2}^2\varepsilon_{2,Z} \quad \dots\dots (14)$$



$$\left.\begin{array}{ll} [E_y match] & \varepsilon_1 + \varepsilon_3 = \varepsilon_2 \\ [H_X match] & n_1(-\cos\alpha\,\varepsilon_1 + \cos\gamma\,\varepsilon_3) = n_2(-\cos\beta\,\varepsilon_2) \\ [H_Z\, match] & n_1(\sin\alpha\,\varepsilon_1 + \sin\gamma\,\varepsilon_3) = n_2(\sin\beta\,\varepsilon_2) \end{array}\right] \dots. (15)$$

However, in light of the reflection law $\gamma = \alpha$ and the Snell's law $n_1 \sin\alpha = n_2 \sin\beta$, the third equation here is equivalent to the first, so there are only 2 independent equations for the two unknown amplitudes ε_2 and ε_3, namely

$$\varepsilon_2 - \varepsilon_3 \ = \varepsilon_1 \ \ and \ \ \frac{n_2 \cos\beta}{n_1 \cos\alpha}\varepsilon_2 + \varepsilon_3 = \ \varepsilon_1 \ldots\ldots\ldots (16)$$

Beside the reflection and the transmission coefficients governing the respective waves' amplitudes, there are related quantities called the reflectivity and the transmissivity which compare the intensities of the reflected / transmitted waves to that of the incident wave. Or rather, they compare the energy flux densities of the respective waves in the ±z direction normal to the boundary, thus

Reflectivity:

$$R = -\frac{(\overrightarrow{\varepsilon_3}^{*} \times \overrightarrow{H_3})_z}{(\overrightarrow{\varepsilon_1}^{*} \times \overrightarrow{H_1})_z} \quad \ldots\ldots\ldots (17)$$

And transmissivity

$$T = +\frac{(\overrightarrow{\varepsilon_2}^{*} \times \overrightarrow{H_2})_z}{(\overrightarrow{\varepsilon_1}^{*} \times \overrightarrow{H_1})_z} \quad \ldots\ldots\ldots (18)$$

by the energy conservation, they should always add up to one,

$$R + T = 1 \ldots\ldots\ldots (19)$$

$$R = \frac{|\overrightarrow{\varepsilon_3}|^2}{|\overrightarrow{\varepsilon_1}|^2} = \ |r|^2 \ldots\ldots\ldots (20)$$

For the EM waves polarized normally to the plane of incidence, the reflectivity is:

$$R = \frac{(\tan\alpha - \tan\beta)^2}{(\tan\beta + \tan\alpha)^2} = \frac{(\cos\alpha - \sqrt{(n_2/n_1)^2 - sin^2\alpha})^2}{(\cos\alpha + \sqrt{(n_2/n_1)^2 - sin^2\alpha})^2} \ldots\ldots\ldots (21)$$

and it is easy to verify that indeed R + T = 1. For general incidence angle α these formulae look somewhat messy, but for the waves hitting the boundary head on (α = 0) they become rather simple:

$$R(\alpha = 0) = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \ldots\ldots\ldots (22)$$