**Groups Theory**

**References:**

* Introduction to Modern Abstract Algebra, by David M. Burton.
* Groups and Numbers, by R. M. Luther.
* A First Course in Abstract Algebra, by J. B. Fraleigh.
* Group Theory, by M. Suzuki.
* Abstract Algebra Theory and Applications, by Thomas W. Judson.
* Abstract Algebra, by I. N. Herstein.
* Basic Abstract Algebra, by P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul.

1. **Definition and Examples of Groups.**

**Definition(1-1):**

A set is a group if it is satisfying the following four axioms

1. a binary operation (**closure)**
2. (**associativity**),
3. s.t.
4. s.t. (**inverse**)

**Examples(1-2):**

1. is a group.

**Solution:** , we have

, ii. iii. , iv.

is a group.

is a group.

**Solution:** i, ii are clear,

iii. ,

iv.

is a group.

**Solution:** i, ii are clear, iii., iv.

5. is a group.

**Solution:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | ? | ? | ? | ? | ? | ? |
|  | ? | ? | ? | ? | ? | ? |
|  | ? | ? | ? | ? | ? | ? |
|  | ? | ? | ? | ? | ? | ? |

We note that axioms i, ii and iii from above table are satisfy axiom iv.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  | ? | ? | ? | ? | ? | ? |

6. is not a group.

**Solution**: since

7. is a group.

**Solution**:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | ? | ? |
|  | ? | ? |

8. Let be a set. Define a binary operation on by the following table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Show that is a group.

**Solution**: axioms i,ii are satisfy from above table, iii. The identity element is , axiom iv.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | ? | ? | ? | ? |

9. is a group.

**Solution**:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |

10. Let , show that is a group.

**Solution**:, we have i. ,

ii. ,

iii.

iv.

11. Let with s.t. are mappings on s.t. . Show that is a group.

**Solution**:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |
|  | ? | ? | ? | ? |

12. Let and be defined by . Show that is a group.

**Solution**: i.

ii.,

iii. ,

iv.

13. Let be an arbitrary group, the set of the functions from into with the composition is forms a group, where s.t. .

**Solution:** i. Let

ii.

iii. is an identity of , since

iv. the inverse of in is , since

14. Let be a positive integer and take , then is an abelian group.

**Definition(1-3):**  A group is an abelian if .

**Example(1-4):** Determine whether the previous examples are abelian .

**Exercises:**

1. Determine whether an abelian group.

* s.t.
* where
* s.t.

1. Show that, is a group.
2. Show that, is an abelian group.