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قسم الفيزياء

Mustansiriyah University

College of science

Physics department

Optics

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Lecture (2-4)

For 3rd year Students

Lecture Title: Reflection and Refraction of Light Using Plane Surfaces and Prisms

Edited by

Prof. Dr. Ali A. Al – Zuky

Refractive index:

The refractive index of a medium is defined as the ratio of velocity of light in a vacuum to the velocity of light in the medium. Refractive index defined as above is called as absolute refractive index. Thus:

$$\mu = n = \frac{c}{v} \quad ....(1)$$

The refractive index is sometimes referred to as optical density. A medium with a relatively high refractive index is said to have a high optical density, while one with a lower index is said to have a low optical density

Note: The refractive index depends not only on the substance but also on the wavelength of the light. The dependence on wavelength is called dispersion i.e($\mu(\boldsymbol{\lambda})$).

Optical path

To derive one of the most fundamental principles in geometric optics, it is appropriate to define a quantity called the optical path. The path L of a ray of light in any medium is given by the product velocity times time:

$$L = vt \quad ...(2)$$

Since by definition $\mu = n = \frac{c}{v}$ which gives $v = \frac{c}{\mu}$we can write:

$$L = \frac{c}{\mu}t \qquad or \qquad \mu L = ct$$

The product μL is called the optical path Δ:

$$\Delta = \mu L \quad ....(3)$$

i.e., Optical path length(Δ) = (Refractive index(μ))(Geometric path length(L)) Thus, the optical path length is defined as the product of refractive index and the geometric path length.

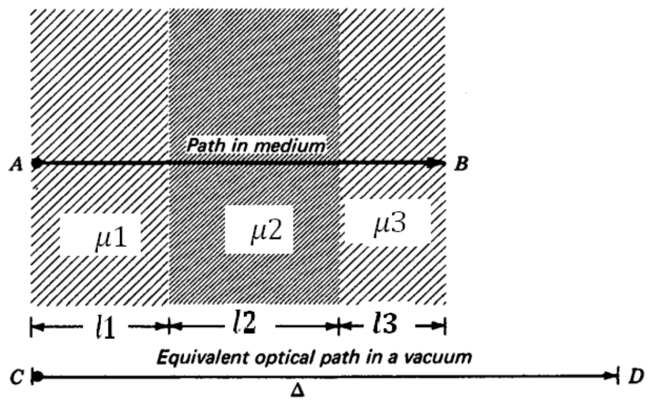
Suppose light travels a distance $l1$ in a medium of refractive index $\mu 1$ (and a distance $l2$ in a medium of refractive index $\mu 2$ in time t. Then:

$$\mu 1 \boldsymbol{l1} = \mu 2 \boldsymbol{l2} \qquad \text{at same time interval(t)}$$

The optical path Δ represents the distance light travels in a vacuum in the same time it travels a distance L in the medium. If a light ray travels through a series of optical media of thickness

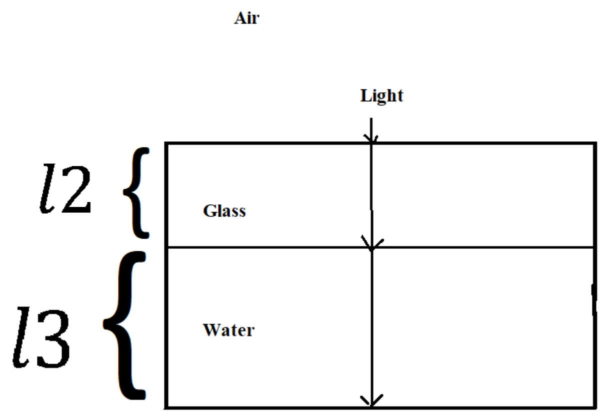
$l1, l2, l3,$ ....and refractive indices $\mu1, \mu2, \mu3, \ldots$.. the total optical path is just the sum of the separate values:

$$\Delta = \mu L = \mu1 l1 + \mu2 l2 + \mu3 l3 + \cdots \qquad ..(4)$$



The optical path through a series of optical media.

Problem: An optical path consists of three media: air, glass, and water. Light travels from air into the glass, then into the water. Given the following information, calculate the total optical path length.

The refractive index of air ($\mu1 = 1$), refractive index of glass ($\mu2 = 1.5$), refractive index of water ($\mu3 = 1.33$), The thickness of the glass ($\boldsymbol{l2 = 2cm}$),and The thickness of the water ($\boldsymbol{l3 = 3cm}$).	

Sol.

The total optical path length (Δ) for light passing through multiple media can be calculated using the formula:

$$\Delta = \Delta1 + \Delta2 + \Delta3 = \mu1 l1 + \mu2 l2 + \mu3 l3$$

In this case, we need to calculate the Δ as light passes through air, glass, and water. In air ($\mu1 = 1, l1 = 0,$ since there's no thickness of air) :

$$\Delta = 1 \times 0 + 1.5 \times 2 + 1.33 \times 3 = 0 + 3 + 3.99,$$

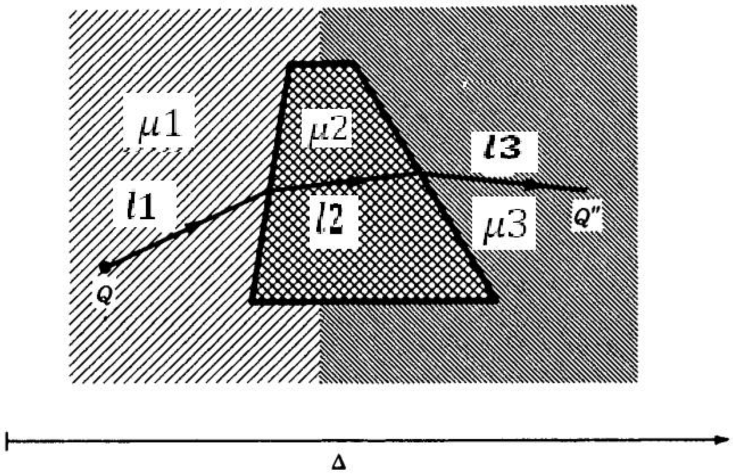
$$\therefore \Delta = 6.99\ cm$$

So, the total optical path length as light passes through air, glass, and water is 0.07 meters.

H.w.: solve previous problem when $l1 = 7cm$

The optical path from the point Q in medium $\mu 1$, through medium $\mu 2$, and to the point Q" in medium $\mu 3$ is given by

$$\Delta = \Delta 1 + \Delta 2 + \Delta 3 = \mu 1 l1 + \mu 2 l2 + \mu 3 l3$$



The refraction of light by a prism and the meaning of optical path Δ.

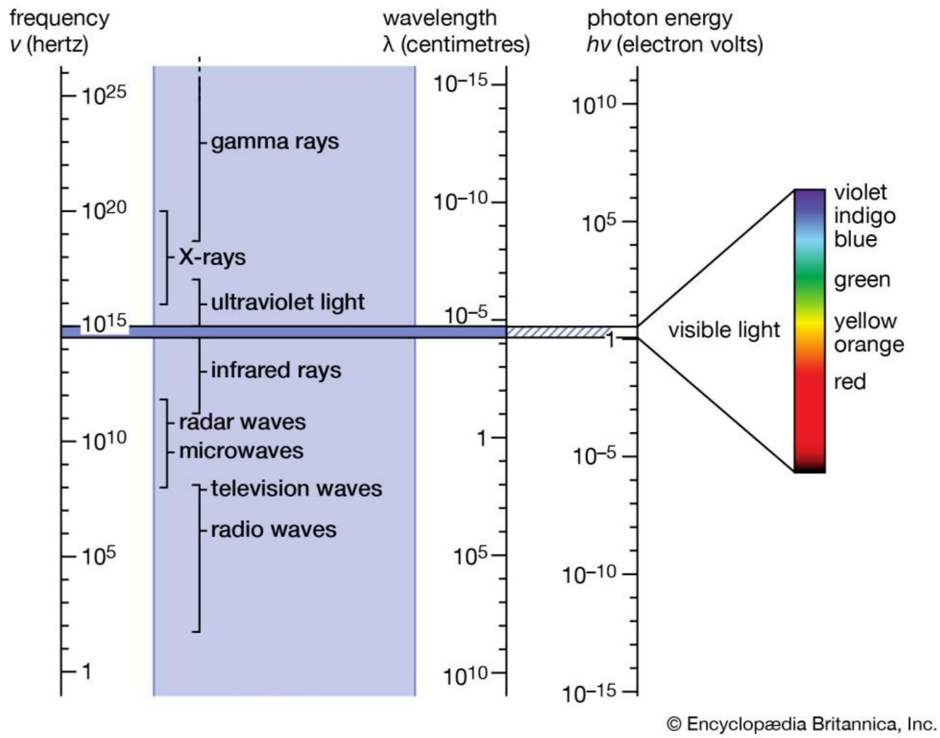
Fermat's. principle:

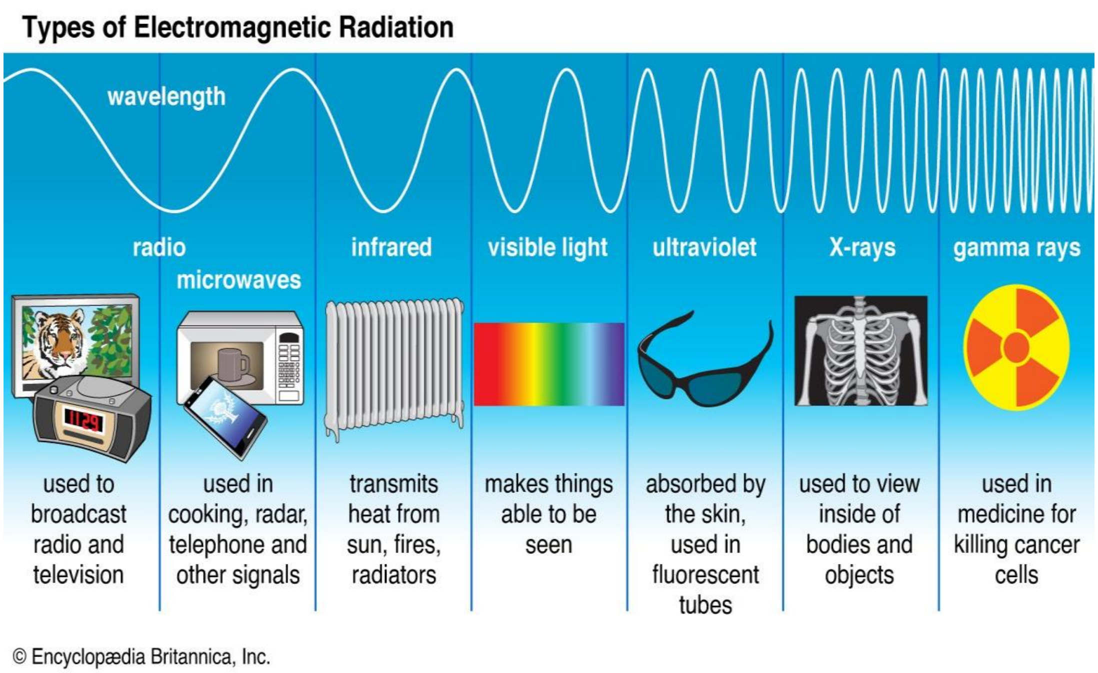
The path taken by a light ray in going from one point to another through any set of media is such as to render its optical path equal, in the first approximation, to other paths closely adjacent to the actual one. Or represent : **The law of shortest times to determine the path of light between two points.** ***When a light ray travels between two points P and Q, it follows, out of all possible paths from P to Q, a path, which requires the least time.***

Electromagnetic waves and visible range:

The arrangement of the various electromagnetic waves in a continuous sequence of frequencies and wavelengths is called an electromagnetic spectrum.

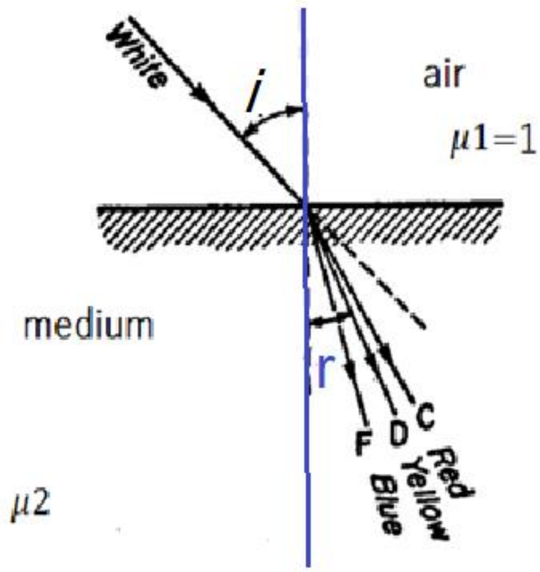
- It is bounded at one end by the gigantic radio waves having wavelengths of a few kilometers and at the other end by y- rays of tiny wavelengths of the order of 10^{-12}m.
- Visible range is that part of the spectrum constituted by waves, which can be detected by the human eye. It extends from the deepest violet to the deepest red(400nm-700nm).





Color Dispersion

The refractive index is a function of wavelength. Generally, it decreases as the wavelength increases. Consequently, light passing through a material medium will be separated according to wavelength. This is known as chromatic dispersion:



Upon refraction, white light is spread out into a spectrum. This is called dispersion.

Spectrum, which is measured by the angle through which ray D is bent. To take a typical case of crown glass, the refractive indices as follows:

Table 1A FRAUNHOFER'S DESIGNATIONS, ELEMENT SOURCE, WAVELENGTH, AND REFRACTIVE INDEX FOR FOUR OPTICAL GLASSES*

Designation	Chemical element	Wavelength, Å†	Spectacle crown	Light flint	Dense flint	Extra dense flint
C	H	6563	1.52042	1.57208	1.66650	1.71303
D	Na	5892	1.52300	1.57600	1.67050	1.72000
F	H	4861	1.52933	1.58606	1.68059	1.73780
G′	H	4340	1.53435	1.59441	1.68882	1.75324

$$\mu F = 1.52933, \mu D = 1.52300, \text{and } \mu c = 1.52042,$$

Now, for a given small angle (i) the dispersion of the F and C rays (r_F - r_c) is proportional to

$$\mu F - \mu c = 0.00891$$

Angular dispersion given by:

$$\Delta r = r_c - r_F$$

While the deviation of the D ray *(i-r_D)* depends on $(\mu D - 1)$ this is equal to 0.52300. The ratio of these two quantities varies greatly for different kinds of glass and it is an important characteristic of any optical substance. It is called the dispersive power and is defined by the equation:

$$V = \frac{\mu F - \mu c}{\mu D - 1} \quad \dots . (5)$$

The reciprocal of the dispersive power is called the dispersive index v:

$$v = \frac{\mu D - 1}{\mu F - \mu c} \quad \dots . (6)$$

For most optical glasses v lies between 20 and 60.

Problem: White light is incident at an angle of i =55° on the polished surface of a piece of glass. If the refractive indices for red C light and blue F light are $\mu c = 1.53828,$ and $\mu F = 1.54735,$ respectively, what is the angular dispersion between these two colors?

Sol.

To find the angular dispersion between red C light and blue F light, you can use the formula:

$$\text{Angular Dispersion} = \Delta r = r_C - r_F \quad \dots . (7)$$

Where: r_F is the angle of refraction for blue F light, and r_c is the angle of refraction for red C light. The formula for the angle of refraction (Snell's law)is:

$$\mu 1 Sin(i) = \mu 2 Sin(r)$$

$$Sin(55) = \mu 2 Sin(r)$$

$$Sin(r) = \frac{Sin(55)}{\mu 2} \quad \dots (8)$$

Where: $\mu 2$ is the refractive index r is the angle of refraction, and $i = 55^{o}$ is the angle of incidence.

First, calculate the angles of refraction for red C and blue F light:

- For red C light (μc = 1.53828) form eq8 get:

$$Sin(r_c) = \frac{Sin(55)}{\mu c} = \frac{0.8192}{1.53828} = 0.5325$$

$$r_c = Sin^{-1}(0.5325) = 32.1774^{o}$$

- For red F light ($\mu F = 1.54735$) form eq8 get:

$$Sin(r_F) = \frac{Sin(55)}{\mu F} = \frac{0.8192}{1.54735} = 0.5294$$

$$r_c = Sin^{-1}(0.5294) = 31.9664^{o}$$

Now, calculate the angular dispersion:

Angular Dispersion = $r_C - r_F \approx 32.1774°$ - $31.9664°$=0.2110°

Angular Dispersion $\Delta r \approx 0.2110°$

So, the angular dispersion between red C light and blue F light is approximately 0.2110°.

Problem: A piece of dense flint glass is to be made into a prism. If the refractive indices for red, yellow, and blue light are specified as μc = 1.64357, μD = 1.64900, and μF = 1.66270. Find the dispersive power and the dispersion constant for this glass.

Sol:

To find the dispersive power (V) and the dispersion constant (υ) for the dense flint glass prism with the given refractive indices, you can use the following formulas:

Dispersive Power = $V = \frac{\mu F - \mu c}{\mu D - 1}$

Dispersion Constant = $\upsilon = \frac{\mu D - 1}{\mu F - \mu c}$

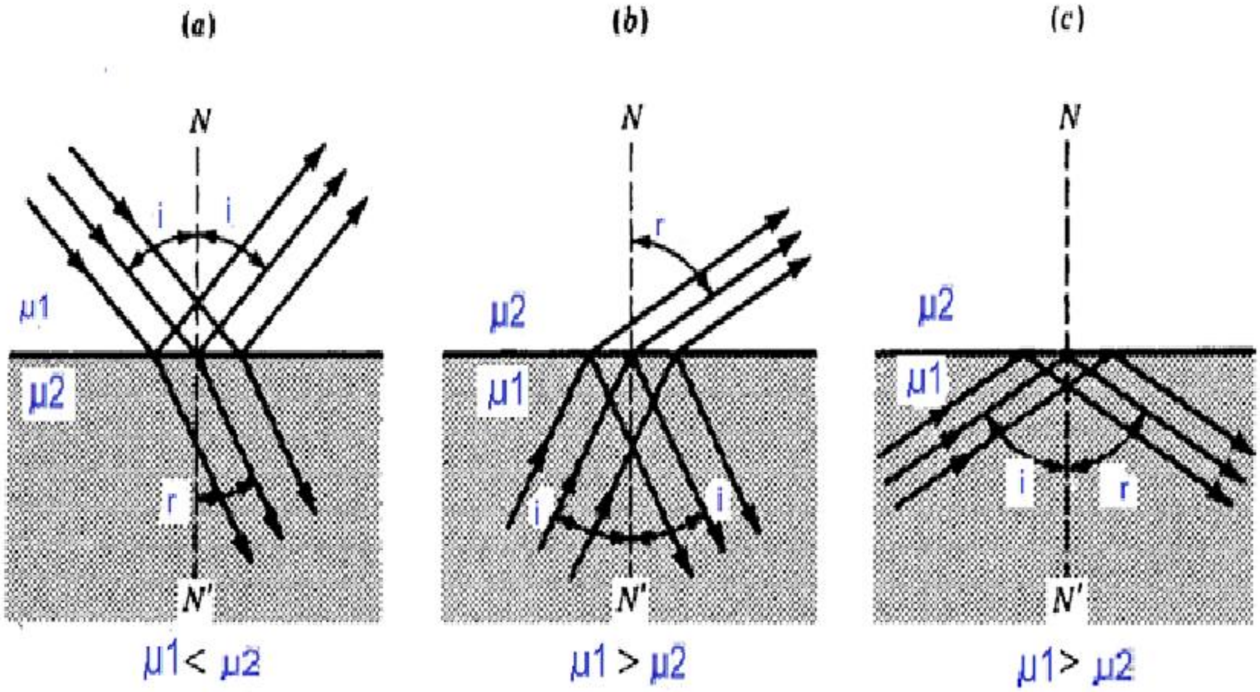
Let's calculate the values:

- Dispersive Power = $V = \frac{\mu F - \mu c}{\mu D - 1} = \frac{1.6627 - 1.64357}{1.649 - 1}$=0.0295
- Dispersion Constant = $\upsilon = \frac{\mu D - 1}{\mu F - \mu c} = \frac{(1.64900 - 1)}{(1.66270 - 1.64357)} = 33.9258$

So, for the dense flint glass prism, the dispersive power is approximately 0.0295, and the dispersion constant is approximately 33.9258.

Plane surfaces:

In a beam or pencil of parallel light, each ray meets the surface traveling in the same direction. Therefore anyone ray may be taken as representative of all the others. The parallel beam remains parallel after reflection or refraction at a plane surface, as shown in following Fig. Refraction causes a change in width of the beam, whereas the reflected beam remains of the same:



Reflection and refraction of a parallel beam: (*a*) external reflection; (*b*) internal reflection at an angle smaller than the critical angle; (*c*) total reflection at or greater than the critical angle.

- Reflection at a surface where μ increases, as in Fig.(a), is called external reflection. It is also frequently termed **rare-to-dense** reflection because the relative magnitudes of μ correspond roughly (though not exactly) to those of the actual densities of materials.
- In Fig(b) is shown a case of internal reflection or **dense-to-rare** reflection. In this particular case the refracted beam is narrow because incidence angle (i) is less than 90°.
- The critical angle and total internal reflection shown in fig(c).

The critical angle and total internal reflection:

Total Internal Reflection (TIR) is a phenomenon in optics that occurs when a light ray traveling from a medium with a higher refractive index to a medium with a lower refractive index strikes the interface (boundary) between the two media at an angle of incidence greater than the critical angle. In this case, instead of being refracted (bent) into the second medium, the light ray is completely reflected back into the first medium:

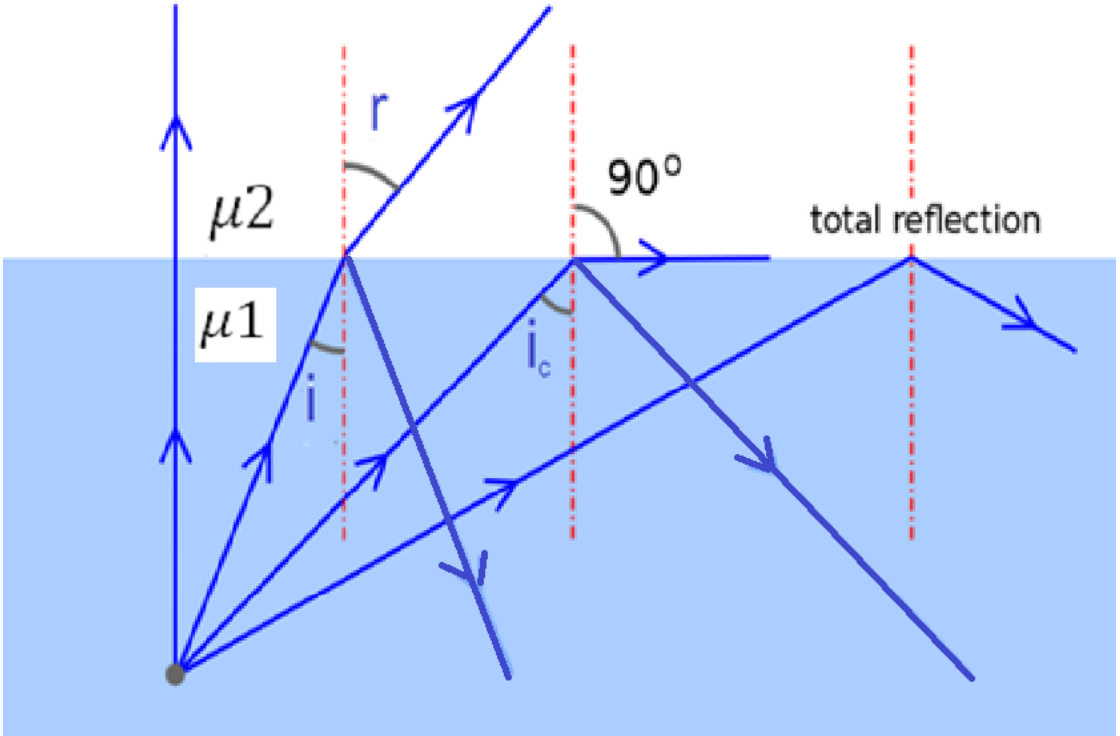


Fig: **Behavior of a ray incident from a medium of higher refractive index $\mu1$ to a medium of lower refractive index $\mu2$, at increasing angles of incidence**

Snell's law: when $\mu1 > \mu2$, become $i < r$.

$$\mu1 \sin(i) = \mu2 \sin(r) \quad \dots (9)$$

$\mu1$ is the refractive index of the medium from which the light is coming (higher refractive index), $\mu2$ is the refractive index of the medium into which the light would refract (lower refractive index). When r=90° eq9 become:

$$\mu1 \sin(i_c) = \mu2 \sin(90) = \mu2$$

$$\sin(i_c) = \frac{\mu2}{\mu1} \quad \dots (10)$$

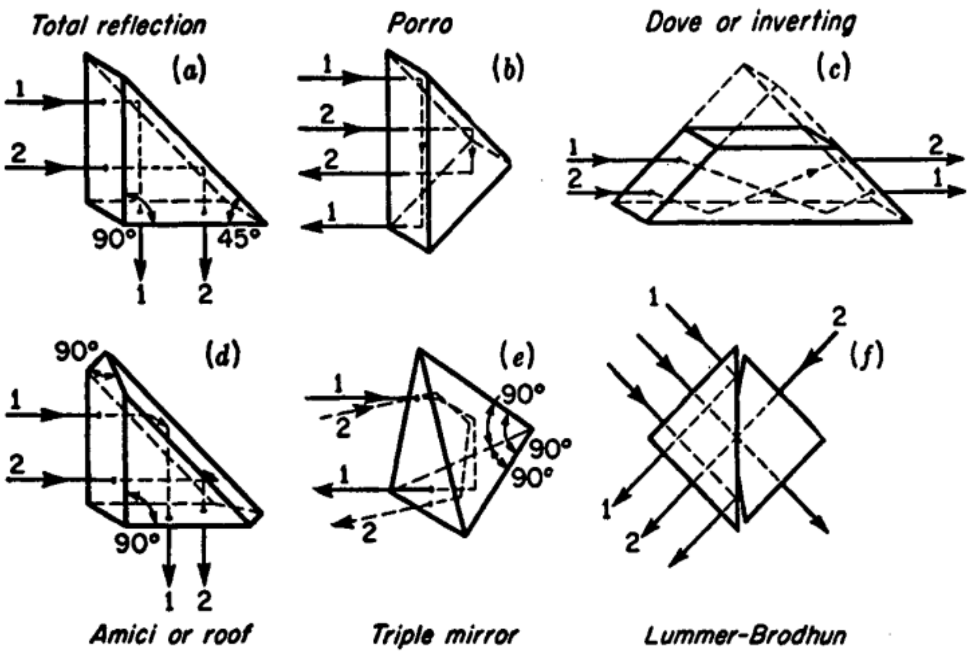
The critical angle (i_c) is the specific angle of incidence at which give refractive angle r=90° and light start totally reflected. It is determined by the refractive indices of the two media involved and is given by the formula:

$$i_c = sin^{-1}\left(\frac{\mu2}{\mu1}\right) \quad \dots (11)$$

If the angle of incidence is greater than the critical angle ($\boldsymbol{i > i_c}$), total internal reflection occurs. Total internal reflection has practical applications in devices like optical fibers and prisms, where light can be transmitted over long distances and efficiently redirected within optical systems.

The critical angle for the boundary separating two optical media is defined as the smallest angle of incidence, in the medium of greater index, for which light is totally reflected.

Many other forms of prisms which use total reflection have been devised for special purposes.



Reflecting prisms utilizing the principle of total reflection.

Plane parallel plate:

When a single ray traverses a glass plate with plane surfaces that are parallel to each other, it emerges parallel to its original direction but with a lateral displacement **d** which increases with the angle of incidence (i) decreases. Using the notation shown in following Fig.

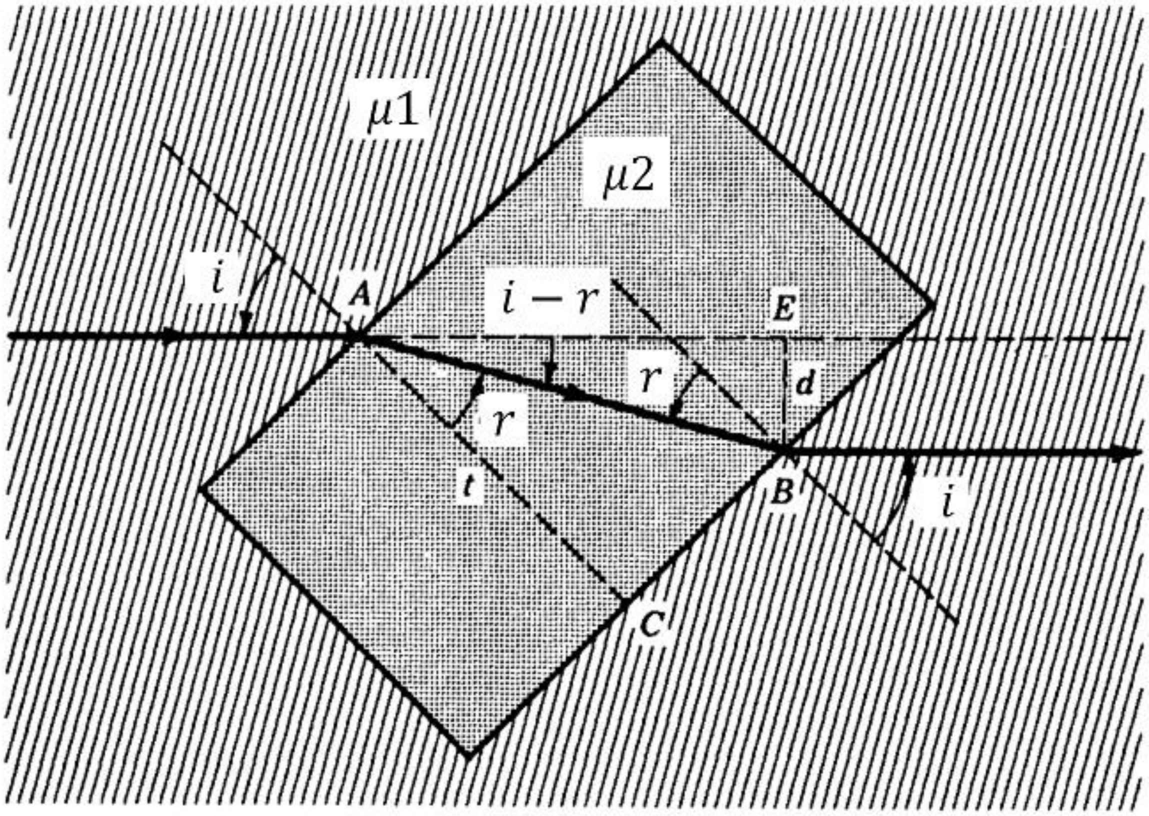


Fig: **Refraction by a plane-parallel plate.**

we may apply the law of refraction and some simple trigonometry to find the displacement d. Starting with the right triangle ABE, we can write:

$$l = AB$$

$$d = l\,sin(i - r) \quad ...(12)$$

which, by the trigonometric relation for the sine of the difference between two angles, can be written:

$$d = l[\sin(i)\cos(r) - \sin(r)\cos(i)] \quad ...(13)$$

From the right triangle ABC we can write: $l = \frac{t}{\cos(r)} \quad ...(14)$

Which, substituted in eq13, get:

$$d = t\frac{\sin(i)\cos(r)}{\cos(r)} - t\frac{\sin(r)\cos(i)}{\cos(r)} \quad ...(15)$$

$$d = t\sin(i) - t\frac{\sin(r)\cos(i)}{\cos(r)} \quad ...(16)$$

From Snell's law we obtain:

$$\sin(r) = \frac{\mu 1}{\mu 2}\sin(i) \qquad \ldots(17)$$

which upon substitution eq17 in eq16, get:

$$d = t[\sin(i) - \frac{\cos(i)}{\cos(r)}\frac{\mu 1}{\mu 2}\sin(i)\,] \qquad \ldots(18)$$

$$d = t\sin(i)\,[1 - \frac{\mu 1}{\mu 2}\frac{\cos(i)}{\cos(r)}] \qquad \ldots(19)$$

From 0° up to appreciably large angles, d is nearly proportional to (i), for as the ratio of the cosines becomes appreciably less than 1, causing the right-hand factor to increase, the sine factor drops below the angle itself in almost the same proportion.

Problem: A light ray is incident on a plane parallel glass plate with a refractive index of 1.5 at an angle of i =30^o to the normal. Calculate the lateral displacement (d) of the ray as it passes through the glass plate, given that the thickness of the glass plate is 1 cm.

Sol:

To find the lateral displacement (d) of the light ray, you can use the formula for lateral displacement when light passes through a thin parallel plate:

$$d = t\sin(i)\left(1 - \frac{\mu 1}{\mu 2}\frac{\cos(i)}{\cos(r)}\right) \qquad \ldots(A)$$

Where: d is the lateral displacement, t=1cm is the thickness of the glass plate, i =30^o is the angle of incidence and r is the angle of refraction unknown. To find r angle used Snell's law:

$$\sin(r) = \frac{\mu 1}{\mu 2}\sin(i)$$

$$\sin(r) = \frac{1}{1.5}\sin(30) = 0.6667 \times 0.5 = 0.3333$$

$$\mathrm{r} = sin^{-1}\,(0.3333) = 19.4692^{o}$$

Now substituting in eq(A) get:

$$d = 1\sin(30)\,(1 - \frac{\mu 1}{\mu 2}\frac{\cos(30)}{\cos(19.4692)})$$

$$d = 0.5\left(1 - \frac{1}{1.5}\frac{\cos(30)}{\cos(19.4692)}\right) = 0.5\left(1 - 0.6667 \times \frac{0.8660}{0.9428}\right)$$
$$= 0.5(1 - 0.6667 \times 0.9185)$$

$$d = 0.5(1 - 0.6124) = 0.5 \times 0.3876$$

$$d = 0.1938cm$$

So, the lateral displacement d of the light ray as it passes through the glass plate is approximately 0.1938cm or 1.938mm.

Refraction by a prism:

In a prism the two surfaces are inclined at some angle α so that the deviation produced by the first surface is not annulled by the second but is further increased. The chromatic dispersion is also increased, and this is usually the main function of a prism. First let us consider, however, the geometrical optics of the prism for light of a single color, i.e., for monochromatic light such as is obtained from a sodium arc.

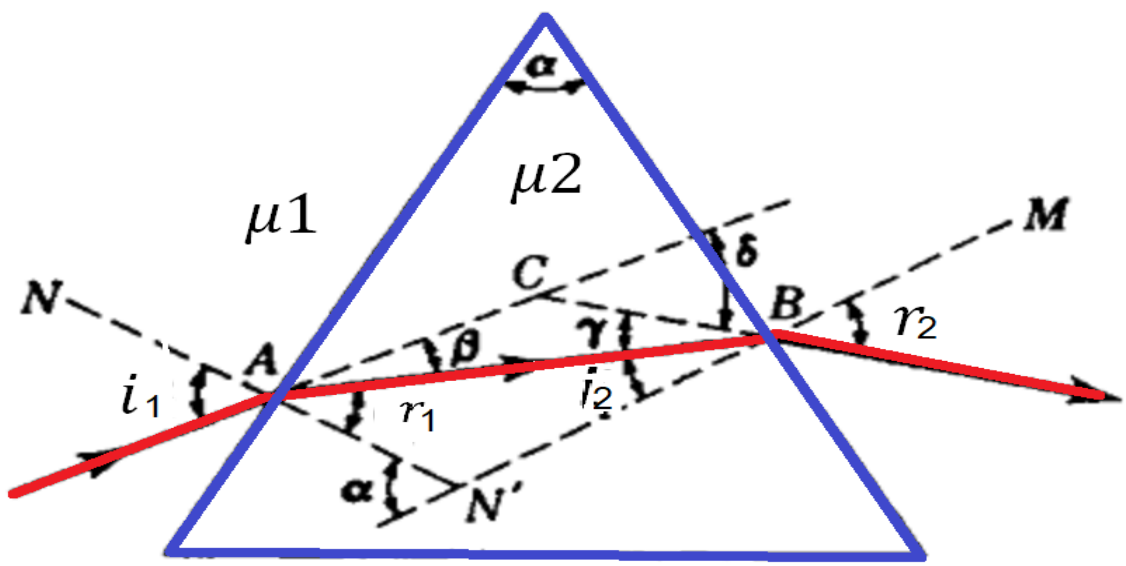


fig: **The geometry associated with refraction by a prism.**

Its refraction at the second surface, as well as at the first surface, obeys Snell's law, so that in terms of the angles shown:

$$\frac{\sin(r1)}{\sin(i1)} = \frac{\mu 1}{\mu 2} = \frac{\sin(i2)}{\sin(r2)} \quad \dots(20)$$

The angle of deviation produced by the first surface is $\beta = i1 - r1$, and that produced by the second surface is $\gamma = i2 - r2$ The total angle of deviation δ between the incident and emergent rays is given by:

$$\delta = \beta + \gamma \quad \dots(21)$$

Since NN' and MN' are perpendicular to the two prism faces, $\alpha$is also the angle at N'. From triangle ABN' and the exterior angle α, we obtain:

$$\alpha = r1 + i2 \quad \dots(22)$$

Combining the above equations, we obtain:

$$\delta = \beta + \gamma = i1 - r1 + r2 - i2$$

$$\delta = (i1 + r2) - (r1 + i2)$$

The prism total angle of deviation δ given by:

$$\delta = i1 + r2 - \alpha \quad \dots(23)$$

The angle of minimum deviation δ_m obtained when:

$$i1 = r2 \ , \qquad i2 = r1, and\ \beta = \gamma$$

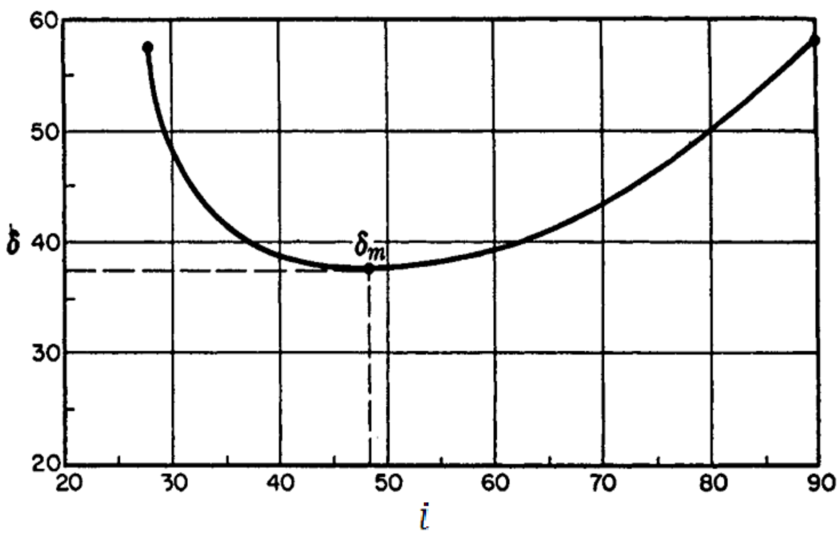


Fig:

A graph of the deviation produced by a 60° glass prism of index μ2 = 1.50. At minimum deviation δ_m = 37.2°, i_1 = 48.6°, and $r1$ = 30.0°.

So the equation of minimum deviation angle given by:

$$\frac{\mu 2}{\mu 1} = \frac{\sin\frac{(\alpha + \delta_m)}{2}}{sin\frac{\alpha}{2}} \quad \dots.(24)$$

Images formed by paraxial rays

Of particular interest to many observers are the object and image distances (**s** and **s'**) sand for rays making small angles i and r.

Rays for which angles are small enough to permit setting the cosines equal to unity and the sines and tangents equal to the angles are called paraxial rays.

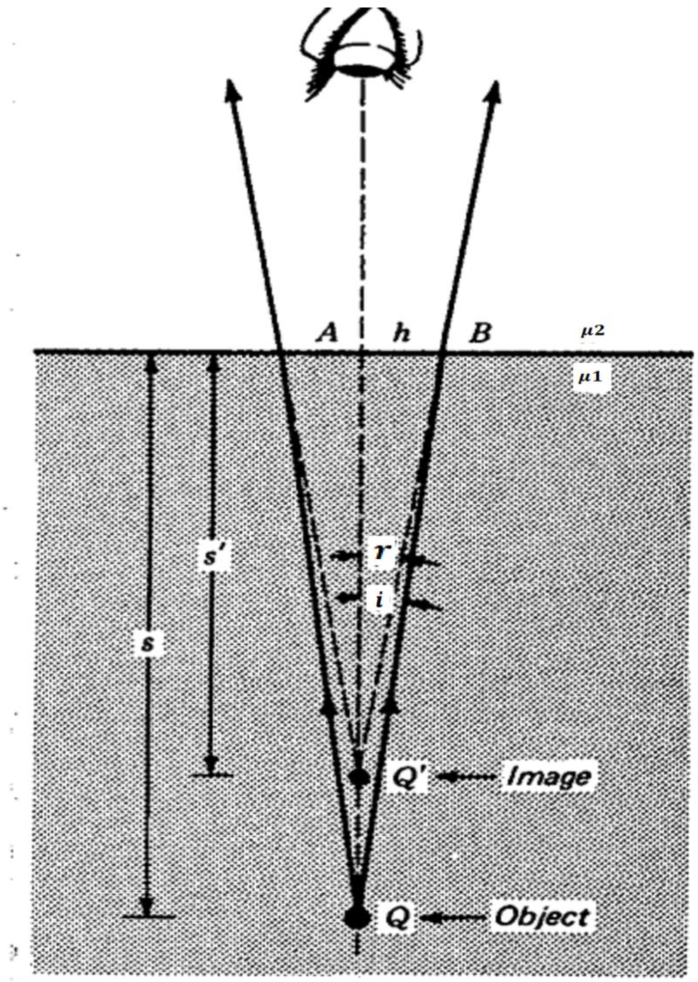


fig: Paraxial rays for an object in water and observed from the air above.

$$s' = s\frac{i}{r} \quad or \quad \frac{s'}{s} = \frac{i}{r} \ ,$$

note i $angle\ of\ object\ ray$

and r $angle\ of\ image\ ray$

$$\frac{i}{r} = \frac{\mu 2}{\mu 1}$$

$$\frac{s'}{s} = \frac{\mu 2}{\mu 1}$$

Utilizing total internal reflection, the British physicist John Tyndall illustrated how light rays within a water tank, passing through an aperture in the side, trace the path of the water flowing from the opening. This phenomenon is frequently observed in modern fountains illuminated by underwater lights.

Fiber optics is a technology that transmits light signals through ultra-thin glass or plastic fibers using total internal reflection. This technology has broad applications, such as high-speed data transmission in telecommunications, medical endoscopy, data networking, sensor technology, and military systems.

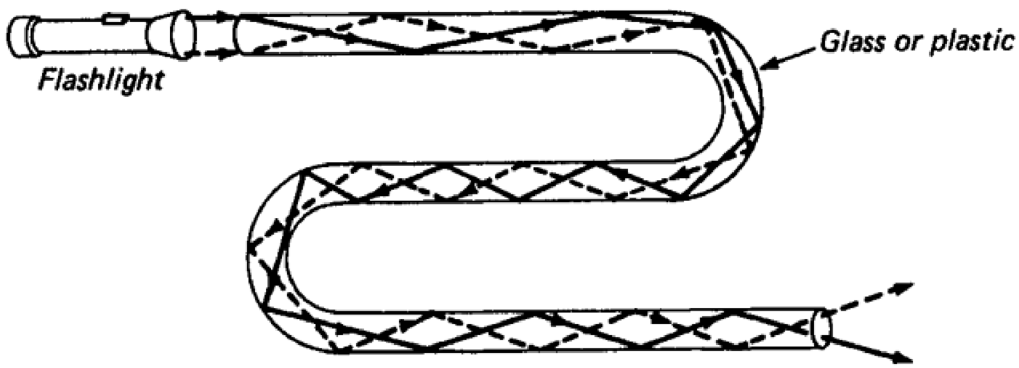


fig:

Light from a flashlight follows a bent transparent rod by total reflection.

Fiber optics

Problem: A 55° prism has a refractive index of 1.68059 for blue light. Determine the angle of deviation for each of the following angles of incidence: 40, 45, 50, 55,60, and 65°.

Sol:

Solve for $i = 40^o$, to determine the angle of deviation for different angles of incidence, you can use Snell's law and the prism's refractive index. The formula for calculating the angle of deviation (δ) is:

$$\delta = i1 + \mathbf{r2} - \alpha$$

Where δ is the angle of deviation, $i1$ *is the a*ngle of incidence for first face, $i2$ is the angle of incidence for the 2nd prism face, and α is the Prism's apex angle

Only $\mathbf{r2}$ **unknown to find** $\mathbf{r2}$ **must be find** $\mathbf{r1}$ ***using Snell's law,***

$$\frac{\sin(r1)}{\sin(i1)} = \frac{\mu 1}{\mu 2} = \frac{\sin(i2)}{\sin(r2)}$$

$$r1 = sin^{-1}\left(\frac{\mu 1}{\mu 2}\sin(i1)\right) = sin^{-1}\left(\frac{1}{1.68059}\sin(40)\right) = sin^{-1}\left(\frac{1}{1.68059}0.64279\right)$$

$$r1 = sin^{-1}(0.3825) = 22.4886^{o}$$

But : $\alpha = r1 + i2$

$$i2 = \alpha - r1 = 55 - 22.4886 = 32.5114^{o}$$

$$\frac{\sin(i2)}{\sin(r2)} = \frac{\mu 1}{\mu 2} \rightarrow \sin(r2) = \sin(i2)\frac{\mu 2}{\mu 1}$$

$$r2 = sin^{-1}\left(\frac{\mu 2}{\mu 1}\sin(i2)\right) = sin^{-1}\left(\frac{1.68059}{1}\sin(32.5114)\right)$$

$$= sin^{-1}(1.68059 \times 0.5375)$$

$$r2 = sin^{-1}(0.9033) = 64.5953^{o}$$

$$\delta = i1 + r2 - \alpha$$

$$\delta = 40 + 64.5953 - 55$$

$$\delta = 49.5953^{o}$$

H.w: when (i = 45, 50, 55,60, and 65°).

References

1. Francis A. Jenkins , and Harvey E.White, “ FUNDAMENTALS OF OPTICS” FourthEdition, McGraw-Hill Higher Education 1981.
2. N. SUBRAHMANYAM, and BRIJ LAL ,”A TEXTBOOK OF OPTICS”, S. CHAND & COMPANY LTD. (AN ISO 9001: 2000 COMPANY) RAM NAGAR, NEW DELHI-110 055, 2000