الجامعة المستنصرية /كلية العلوم

قسم الفيزياء

Mustansiriyah University

College of science

Physics department

Optics

2023-2024

Lecture (5-6)

For 3rd year Students

Lecture Title: Problems, Solutions, and Assignments on Light Refraction

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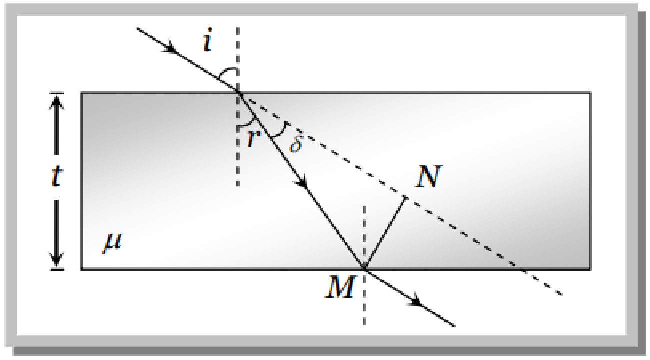
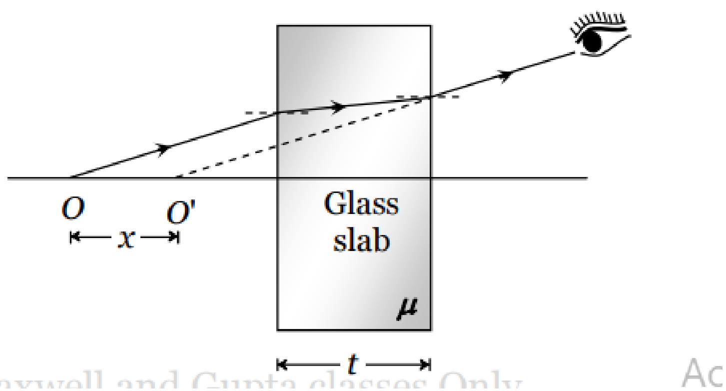
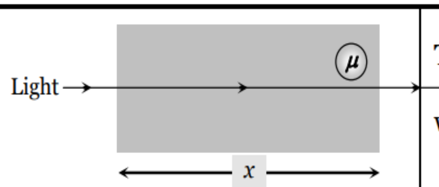
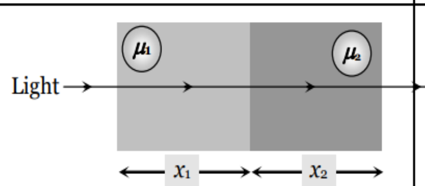
Notes, definitions, and Additional Illustrative Figures:



Normal shift

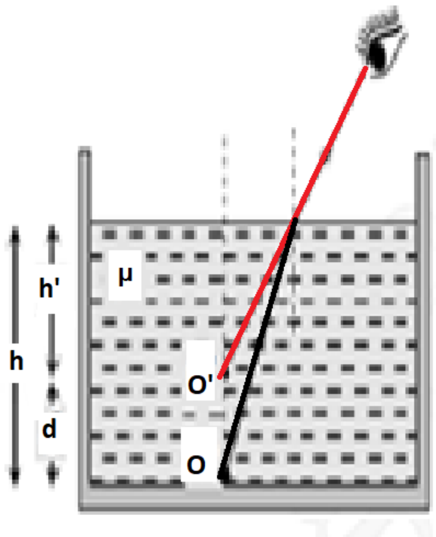
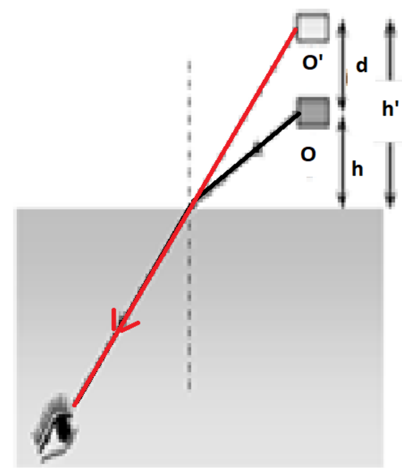


Optical path: It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium.

	Time taken by light ray to pass through the medium $= \frac{\mu x}{c}$; where x = geometrical path and μx = optical path
	For two medium in contact optical path = $\mu_1 x_1 + \mu_2 x_2$

Real and Apparent Depth:

If object and observer are situated in different medium then due to refraction, object appears to be displaced from its real position. There are two possible conditions

When object is in denser medium and observer is in rare medium	When object is in rare medium and observer is in denser medium
	

(2) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{h}{h'}$ Real depth >Apparent depth that's why *a* coin at the bottom of bucket (full of water) appears to be raised)	(2) $\mu = \frac{h'}{h}$ Real depth < Apparent depth that's why high flying aeroplane appears to be higher than it's actual height.
(3) Shift $d = h - h' = \left(1 - \frac{1}{\mu}\right)h$	(3) $d = (\mu - 1)h$
(4) For water $\mu = \frac{4}{3} \Rightarrow d = \frac{h}{4}$ For glass $\mu = \frac{3}{2} \Rightarrow d = \frac{h}{3}$	(4) Shift for water $d_w = \frac{h}{3}$ Shift for glass $d_g = \frac{h}{2}$

- Greater the refractive index lesser will be the critical angle.

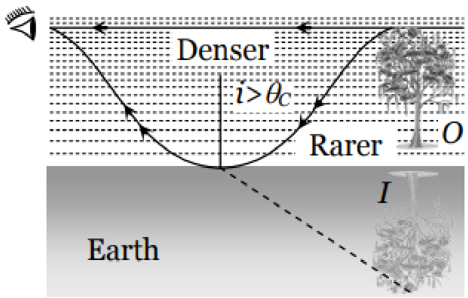
  (b) For (water-air) pair μ=1.33 → $i_{c\ water} = 49.19$

  (a) For (glass- air) pair μ=1.5→ $i_{c\ glass} = 41.81$

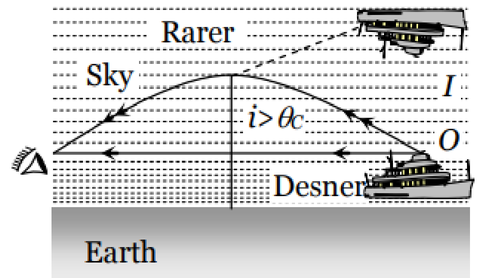
  (c) For (diamond-air) pair μ=2.42→ $i_{c\ diamond} = 24.42$
- With temperature rise refractive index of the material decreases therefore critical angle increases

Examples of total internal reflection (TIR)

1. Brilliance of diamond: Due to repeated internal reflections diamond sparkles.
2. Mirage and looming.



Mirage : An optical illusion in deserts



Looming : An optical illusion in cold countries

3. Optical fiber : Optical fibers consist of many long high quality composite glass/quartz fibers. Each fiber consists of a core and cladding. The refractive index of the material of the core (μ1) is higher than that of the cladding (μ2).
4. Field of vision of fish (or swimmer) : A fish (diver) inside the water can see the whole world through a cone with:

a) Apex angle $= 2i_c = 2C = 98^o$

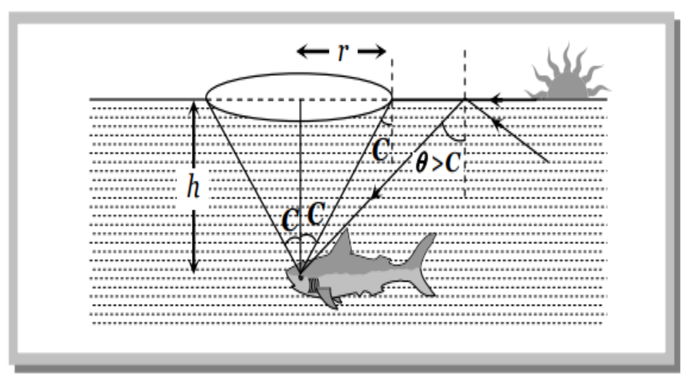
b) Radius if base $r = h\tan(i_c) = \frac{h}{\sqrt{\mu^2-1}}$

c) Area of base $A = \frac{\pi h^2}{\mu^2-1}$

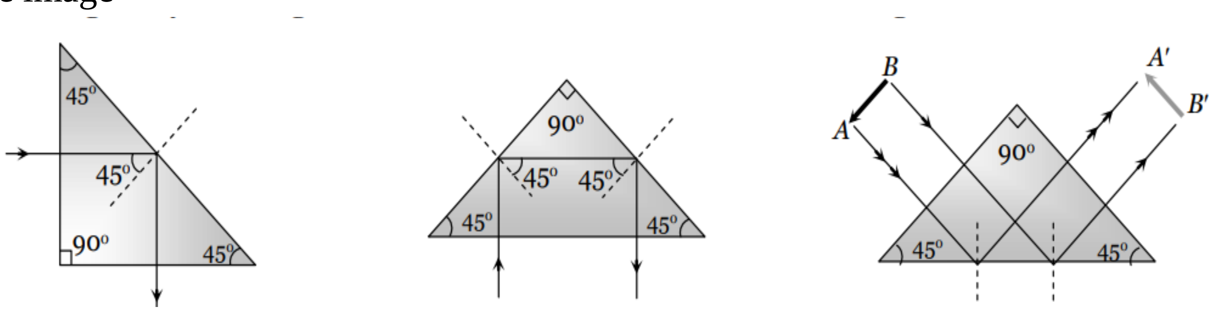
For water

$$\mu = 1.33 = \frac{4}{3}\ so\ r = \frac{3h}{\sqrt{7}}, and\ A = \frac{9\pi h^2}{7}$$

$$i_c = C$$

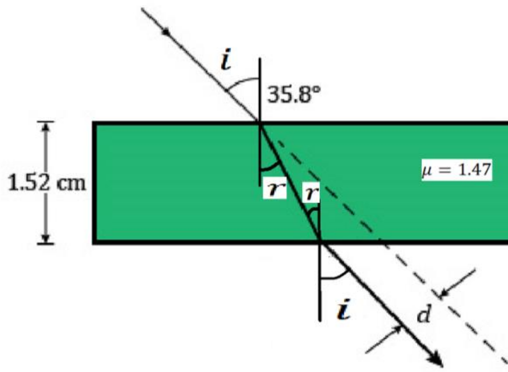


5. Porro prism : A right angled isosceles prism, which is used in periscopes or binoculars. It is used to deviate light rays through o 90 and o 180 and also to erect the image



Solved Problems for previous lectures:

Problem1: A beam of light passes through a plate of glass with a thickness of 1.52cm and a refractive index $\mu = 1.47$. The beam enters from the air at an angle of 35.8^o. Determine the deviation d of the ray as illustrated in below diagram:.



sol:

used $d = l\, sin(i - r)$ ... $(12lec2\_4)$ and $l = \frac{t}{\cos(r)}$ ... $(14lec2\_4)$ $to\ get$:

$$d = t\frac{sin(i - r)}{\cos(r)} \quad ...$$

Or used

$$d = t\sin(i)\left[1 - \frac{\mu 1}{\mu 2}\frac{\cos(i)}{\cos(r)}\right] \quad ...(19lec2\_4)$$

Must be find angle of refraction (r) using Snell's law:

$$\sin(r) = \frac{\mu 1}{\mu 2}\sin(i)$$

$$\sin(r) = \frac{1}{\mu}\sin(i)$$

$$\sin(r) = \frac{1}{1.47}\sin(35.8) = \frac{0.5851}{1.47} = 0.3980$$

$$r = sin^{-1}(0.398) = 23.4532^{\circ}$$

$$\therefore d = t\frac{sin(i - r)}{\cos(r)}$$

$$d = 1.52\frac{sin(35.8 - 23.4532)}{\cos(23.4532)} = 1.52\frac{sin(12.3468)}{\cos(23.4532)} = 1.52\frac{0.2138}{0.9174}$$

$$d = 1.52 \times\ 0.2330 =\ 0.3542\ cm$$

Problem2: A laser beam is directed through a rectangular block of acrylic glass (PMMA) with a refractive index (μ) of 1.49. The incident angle of the laser beam is 45°. The block has a thickness of 2.5cm. Calculate the lateral displacement (deviation) of the laser beam as it exits the acrylic glass block.

Sol:

$$\sin(r) = \frac{\mu 1}{\mu 2}\sin(i)$$

$$\sin(r) = \frac{1}{\mu}\sin(i)$$

$$\sin(r) = \frac{1}{1.49}\sin(45) =\ 0.4746$$

$$r = sin^{-1}(\ 0.4746) = 28.3333^{o}$$

$$\therefore\ d = t\frac{sin(i - r)}{\cos(r)}$$

$$d = 2.5\frac{sin(45 - 28.3333)}{\cos(28.3333)} = 2.5\frac{sin(\ 16.6667)}{\cos(28.3333)} = 2.5\frac{0.2868}{0.8802}$$

$$d = 2.5 \times\ \ 0.3258 =\ 0.8145\ cm$$

Problem3: A ray of light is incident on a piece of glass at an angle of 45°. If the angle of refraction is 25.37°, find (a) the refractive index and (b) the critical angle:

Sol:

we'll use Snell's Law for refraction, which relates the angle of incidence (i), the angle of refraction (r), and the refractive indices of the two media. Snell's Law is given by:

$$\mu 2\sin(r) = \mu 1\sin(i)$$

$$\mu 2 = \mu 1\frac{sin(i)}{\sin(r)} = 1\frac{sin(45)}{\sin(25.37)} = \frac{0.7071}{0.4285} =\ 1.6502$$

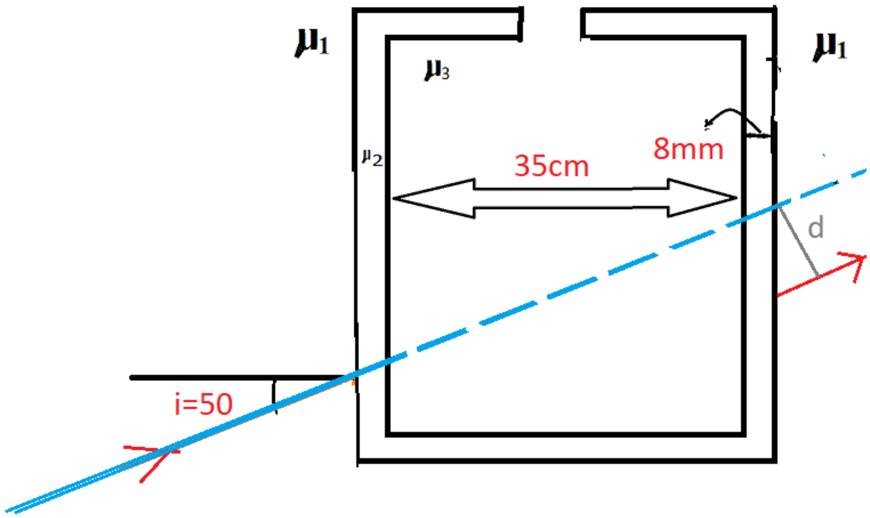
$$i_{c} = sin^{-1}\left(\frac{1}{\mu} \right)\ \ \ldots(11)lect2\_4$$

The critical angle for light transitioning from glass to air is as follows:

$$i_c = sin^{-1}\left(\frac{1}{1.6502}\right) = sin^{-1}\left(\frac{1}{1.6502}\right) = sin^{-1}(0.6060) = 37.3008^o$$

Problem4: A rectangular aquarium is to be filled with water. The sides are made of glass plates 8 mm thick. Inside, the walls are 35cm apart, and the refractive index of the glass is1.5250. If a ray of light is incident on one side at an angle of 50°, find the lateral displacement produced when the tank is (a) empty and (b) filled with water.

Sol:



Case 1:

$\mu 3 = \mu 1$ no water in tank

Must be find angle of refraction (r) using Snell's law:

$$\sin(r) = \frac{\mu 1}{\mu 2}\sin(i)$$

$$\sin(r) = \frac{1}{\mu}\sin(i)$$

$$\sin(r) = \frac{1}{1.5250}\sin(50) = \frac{0.7660}{1.525} = 0.5023$$

$$r = sin^{-1}(0.5023) = 30.1523^o$$

$\therefore d1 = t\frac{sin(i-r)}{\cos(r)}$ form 1st face

$$d = 0.8\frac{sin(50-30.1523)}{\cos(30.1523)} = 0.8\frac{sin(19.8477)}{\cos(30.1523)} = 0.8\frac{0.3395}{0.8647}$$

$$d1 = 0.8 \times 0.3926 = 0.3141\ cm$$

At the same way compute the deviation d2 from the 2nd face, so:

$$d2 = 0.3141\ cm$$

$$\therefore d = d1 + d2 = 0.3141 + 0.3141 = 0.6282cm$$

In case 2: tank fills with water:

$$\mu 3 = 1.33\ \neq \mu 1$$

In same way compute d1 in this case air-glass: t1=0.8cm glass thickness

$$d1 = 0.3141\ cm$$

The deviation caused by water d2 computed as follows:

Here t2=35cm tank thickness, and i2=r1=30.1523°

$$\mu 2 \sin(i2) = \mu 3 \sin(r2)$$

$$\sin(r2) = \frac{\mu 2}{\mu 3} \sin(i2)$$

$$\sin(r2) = \frac{1.525}{1.33} \sin(30.1523) = 1.1466 \times\ 0.5023 =\ 0.5759$$

$$r2 = sin^{-1}(\ 0.5759) = 35.1627^{o}$$

$$\therefore d2 = t2 \frac{sin(i2 - r2)}{\cos(r2)}$$ form water

$$d2 = 35 \frac{sin(30.1523 - 35.1627)}{\cos(35.1627)} = 35 \frac{sin(-5.0104)}{\cos(35.1627)} = 35 \frac{-0.0873}{0.8175}$$

$$d2 = 35 \times (-0.1068) = -3.7380\ cm$$

Now compute the deviation caused by 2nd face of tank d3 as follows:

Here t3=0.8cm glass thickness and i3=r2=35.1627°

$$\mu 3 \sin(i3) = \mu 2 \sin(r3)$$

$$\sin(r3) = \frac{\mu 3}{\mu 2} \sin(i3)$$

$$\sin(r3) = \frac{1.33}{1.525} \sin(35.1627) = 0.8721 \times\ \ 0.5759 =\ 0.5022$$

$$r3 = sin^{-1}(\ 0.5022) = 30.1457^{o}$$

$$\therefore d3 = t3 \frac{sin(i3 - r3)}{\cos(r3)}$$ form 2nd face of tank

$$d3 = 0.8 \frac{sin(35.1627 - 30.1457)}{\cos(30.1457)} = 0.8 \frac{sin(5.0170)}{\cos(30.1457)} = 0.8 \frac{0.0875}{0.8648}$$

$$d3 = 0.8 \times 0.1012 = 0.0810\ cm$$

So the total deviation given by:

$$d = d1 + d2 + d3 = 0.3141 - 3.7380 + 0.0810$$

$$d = -3.3429cm$$

Problem5: A ray of light is incident on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5 at an angle of 60^o with the normal. Calculate the lateral shift produced. (Given sin35.3∘=0.5773, cos35.3∘=0.816, sin24.7∘=0.148).

Sol:

The correct is d=0.0513 given i=60^o thickness of the slab, t=0.1m, refractive index of the slab $\mu = 1.5$. let the angle of refraction angle is r1. We know that

$$\mu = \frac{\sin(i1)}{\sin(r1)}$$

$$\sin(r1) = \frac{\sin(i1)}{\mu} = \frac{\sin(60)}{1.5} = 0.5773$$

$$r1 = sin^{-1}(0.5773 = 35.5^o$$

The lateral shift (d) by glass slab is given by:

$$d = t\frac{\sin(i1 - r1)}{\cos(r1)} = 0.1\frac{\sin(60 - 35.5)}{\cos(35.5)} = 0.1\frac{\sin(24.7)}{\cos(35.5)}$$

$$d = 0.1\frac{0.418}{0.816}$$

$$d = 0.0513\text{m}$$

Problem6: A beam of monochromatic blue light of wavelength 4200 Å in air travels in water ($\mu = 4/3$). Its wavelength in water will be: (a) 2800 Å (b) 5600 Å (c) 3150 Å (d) 4000 Å

Solution: (c)

$$\mu = \frac{c}{v} = \frac{f_o\lambda_o}{f\lambda}$$

$$but\ f_o = f and\ \lambda_o \neq \lambda$$

$$\therefore \mu = \frac{\lambda_o}{\lambda} \qquad important$$

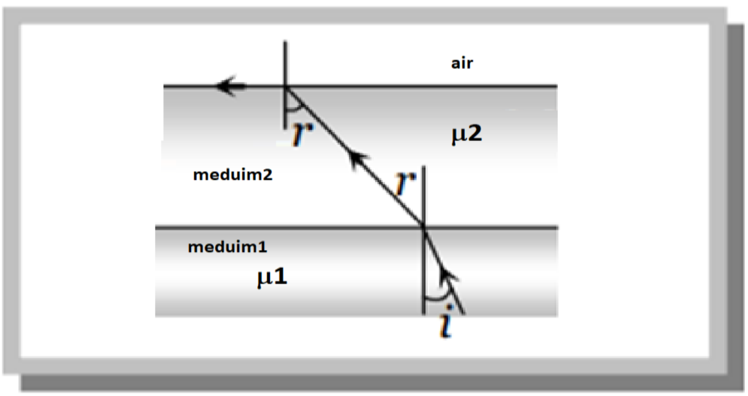
$$\therefore \quad \mu\ \alpha \frac{1}{\lambda}$$

$$\frac{\mu_1}{\mu_2} = \frac{\frac{\lambda_o}{\lambda_1}}{\frac{\lambda_o}{\lambda_2}} = \frac{\lambda_2}{\lambda_1}$$

$$\frac{\mu_1}{\mu_2} = \frac{1}{\frac{4}{3}} = \frac{\lambda_2}{4200}$$

$$\lambda_2 = 4200 \times \frac{3}{4} = 3150A^o$$

Problem7: A ray of light is incident at the meduim1–meduim2 interface at an angle i, it emerges finally parallel to the surface of meduim2. Prove that the value of : $\mu_1 = \frac{1}{\sin(i)}$



Sol:

For medium1-medium2 interface

$$\frac{\mu_1}{\mu_2} = \frac{\sin(r)}{\sin(i)} \quad ..(1)$$

For medium2-air interface:

$$\frac{\mu_2}{\mu_{air}} = \frac{\sin(r2)}{\sin(i2)} = \frac{\sin(90)}{\sin(r)}$$

$$\frac{\mu_2}{1} = \frac{\sin(90)}{\sin(r)}$$

$$\frac{\mu_2}{1} = \frac{1}{\sin(r)}$$

$$\sin(r) = \frac{1}{\mu_2} \quad \dots (2)$$

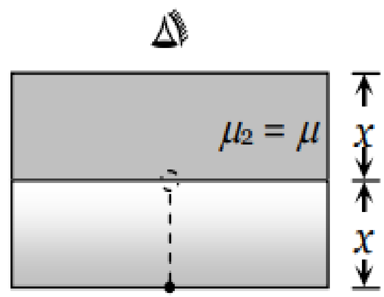
Substitute eq2 in eq1 get:

$$\frac{\mu_1}{\mu_2} = \frac{\sin(r)}{\sin(i)} = \frac{\frac{1}{\mu_2}}{\sin(i1)}$$

$$\frac{\mu_1}{\mu_2} = \frac{\frac{1}{\mu_2}}{\sin(i)}$$

$$\mu_1 = \frac{1}{\sin(i)}$$

Problem8: A rectangular slab of refractive index μ is placed over another slab of refractive index 3, both slabs being identical in dimensions. If a coin is placed below the lower slab, for what value of μ will the coin appear to be placed at the interface between the slabs when viewed from the top: (a) 1.8, (b) 2, (c) 1.5, (d)=2.5.



Sol:

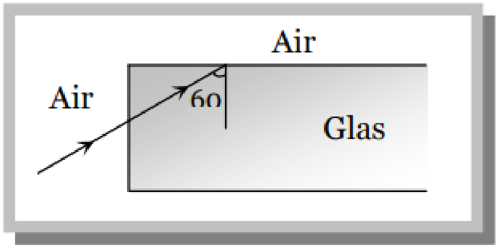
(c) Apparent depth of coin as seen from top:

$$d = \frac{x}{\mu 1} + \frac{x}{\mu 2} = x \rightarrow \frac{1}{\mu 1} + \frac{1}{\mu 2} = 1 \rightarrow \frac{1}{3} + \frac{1}{\mu} = 1$$

$$\frac{1}{\mu} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\mu = 1.5$$

Problem9: A light ray from air is incident (as shown in figure) at one end of a glass fiber making an incidence angle of 60^o on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fiber of length 1 km: a) 3.33μs, b) 6.67μs, c) 5.77μs, d) 3.85μs.



Sol: d)

When total internal reflection just takes place from lateral surface then:

$$i = i_c = C = 60^o$$

From $$\mu = \frac{1}{\sin(i_c)} = \frac{1}{\sin(60)} = \frac{2}{\sqrt{3}}$$

Hence time taken by light transverse some distance in medium:

$$t = \frac{\mu x}{c} = \frac{\frac{2}{\sqrt{3}}(1 \times 10^3)}{(3 \times 10^8)} = 3.85\mu s$$

H.w.

1. A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Prove the angle of incidence is $i = 2\cos(\frac{\mu}{2})$ use relation $\sin(2\theta) = 2\sin(\theta)\, cose(\theta)$
2. Calculate the lateral displacements of rays of light incident on a block of glass with parallel sides of thickness 5cm, at the following angles: **(i) 5°, (ii) 10°, (ii) 15°, (iv) 20°, (v) 30°,and (vi) 40°.**
3. On a glass plate a light wave is incident at an angle of 60^o. If the reflected and the refracted waves are mutually perpendicular, the refractive index of material is:

   .$(a)\frac{\sqrt{3}}{2}$, $(b)\sqrt{3}$, $(c)\frac{3}{2}$, $(d)\frac{1}{\sqrt{3}}$

4. Velocity of light in glass whose refractive index with respect to air is 1.5 is 2×10^{8} m/s and in certain liquid the velocity of light found to be 2.50×10^{8} m /s . The refractive index of the liquid with respect to air is: (a) 0.64, (b) 0.8, (c) 1.2, (d)1.44.

Hinte: $\mu = \frac{c}{v}$, $\rightarrow$ $\frac{\mu_1}{\mu_2} = \frac{\frac{c}{v_1}}{\frac{c}{v_2}} = \frac{v_2}{v_1}$ $\rightarrow \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$

5. The ratio of thickness of plates of two transparent mediums A and B is 6 : 4. If light takes equal time in passing through them, then refractive index of B with respect to A will be :
.(a)1.4 (b) 1.5 (c) 1.75 (d) 1.33.

6. A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Then the angle of incidence is:

$$(a)\cos^{-1}\left(\frac{\mu}{2}\right), (b)2\cos^{-1}\left(\frac{\mu}{2}\right), \quad (c)2\sin^{-1}(\mu), \quad (d)2\sin^{-1}\left(\frac{\mu}{2}\right),$$

7. The wavelength of light in two liquids 'x' and 'y' is 3500 Å and 7000 Å, then the critical angle of x relative to y will be : **(a) 60^{o} (b) 45^{o} (c) 30^{o} (d) 15^{o}** .

References

1. A. Jenkins , and Harvey E.White, " FUNDAMENTALS OF OPTICS" FourthEdition, McGraw-Hill Higher Education 1981.
2. N. SUBRAHMANYAM, and BRIJ LAL ,"A TEXTBOOK OF OPTICS", S. CHAND & COMPANY LTD. (AN ISO 9001: 2000 COMPANY) RAM NAGAR, NEW DELHI-110 055, 2000