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قسم الفيزياء

Mustansiriyah University

College of science

Physics department

Optics

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Lecture (7)

For 3rd year Students

Lecture Title: An Introduction to Light Refraction across Spherical Surfaces and Lenses

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Refraction by Spherical Surface:

Many common optical devices contain not only mirrors and prisms having flat polished surfaces but lenses having spherical surfaces with a wide range of curvatures. Such spherical surfaces, in contrast with plane surfaces treated in the last lectures, are capable of forming real images:

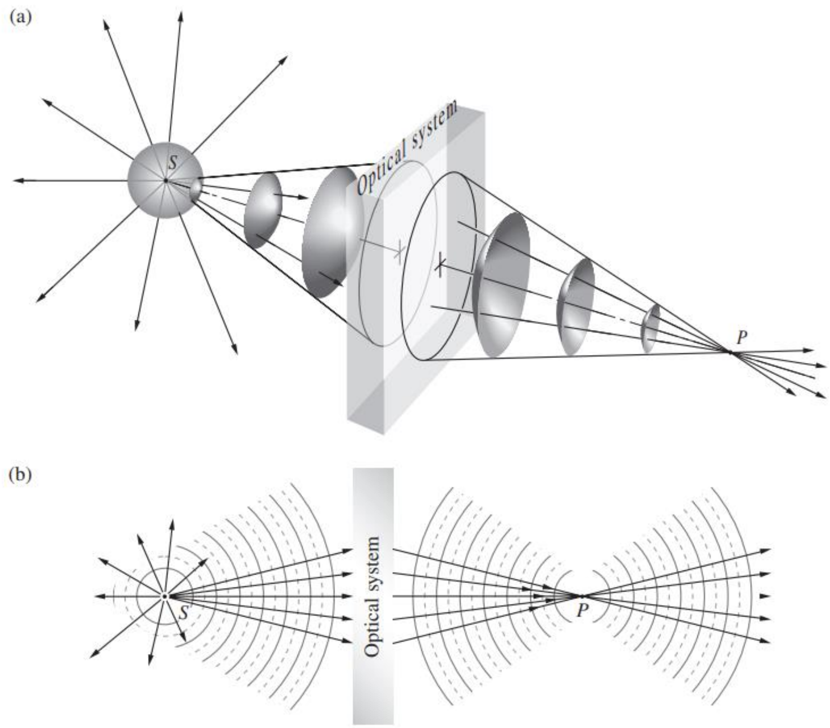
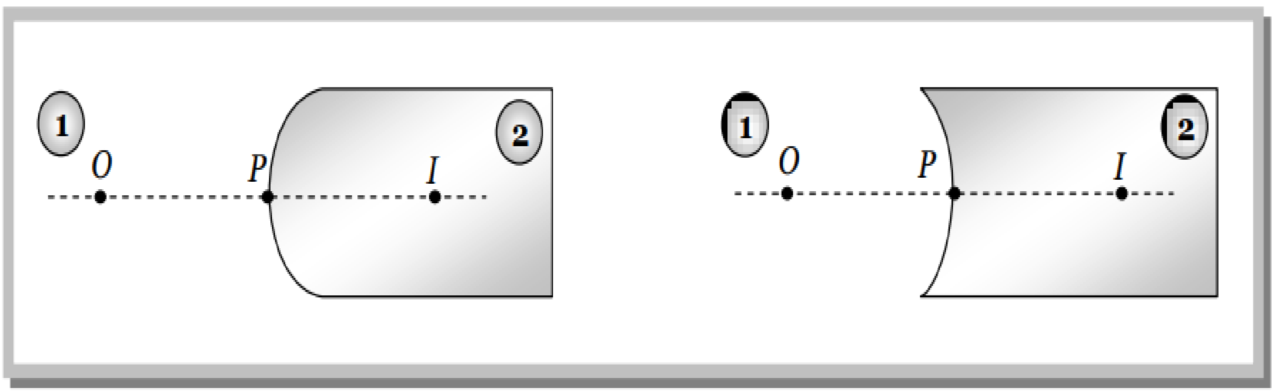


Fig: Conjugate foci. (a) A point source S sends out spherical waves. A cone of rays enters an optical system that inverts the wavefronts, causing them to converge on point-P. (b) In cross section rays diverge from S, and a portion of them converge to P. If nothing stops the light at P, it continues on



$\mu 1$ = Refractive index of the medium from which light rays are coming (from object).

$\mu 2$ = Refractive index of the medium in which light rays are entering.

Focal point and focal length:

The focal length of an optical system serves as a fundamental indicator of its ability to either converge or diverge light.

- A positive focal length signifies the optical system's capability to converge incident light,
- A negative focal length signifies its capacity to diverge light.

Lenses are optical components designed to bend and focus light. They have two fundamental properties related to their focusing ability:

- focal points,
- focal lengths.

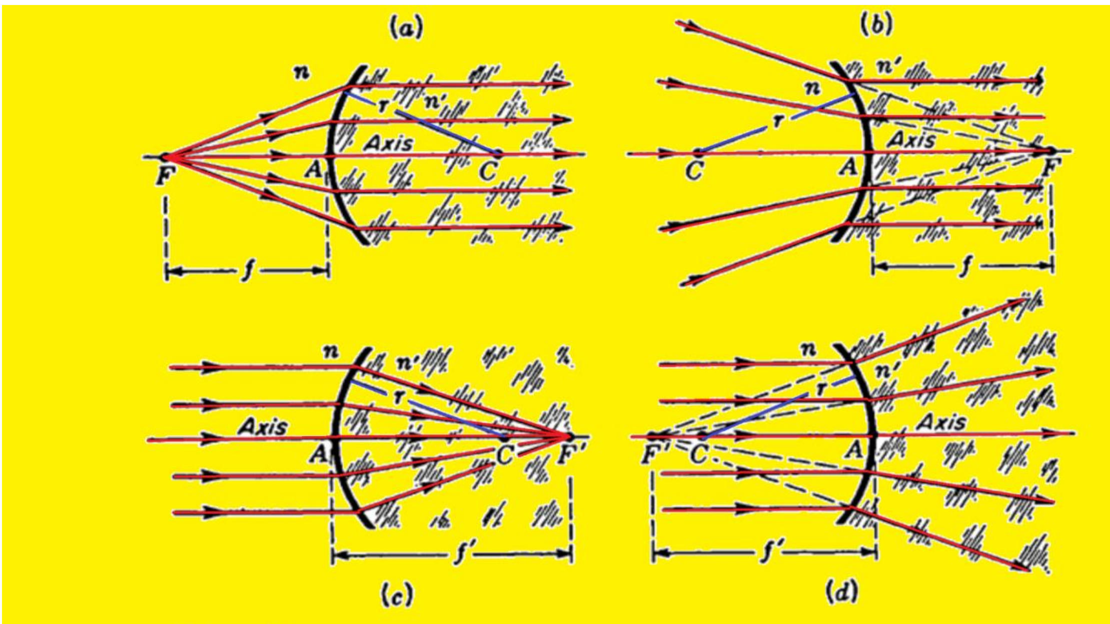


Fig: The focal points F and F' and focal lengths f and f' associated with a single spherical refracting surface of radius r separating two media of index μ and μ'.

Following Figure depicts a wave from the **point source S** impinging on a spherical interface of radius R centered at C. The **point-V** is called **the vertex** of the surface. The length u = SV is known as the **object distance**. The ray **SA** will be refracted at the interface toward the local normal ($\mu 2 > \mu 1$) and therefore toward the **central or optical** axis. Assume that at some **point-P** the ray will cross the axis, as will all other rays incident at the same angle (i). The length v= VP is the **image distance**. Fermat's Principle maintains that the optical path length **OPL** will be stationary:

$$OPL = \mu 1 l_o + \mu 2 l_i \quad \ldots (1)$$

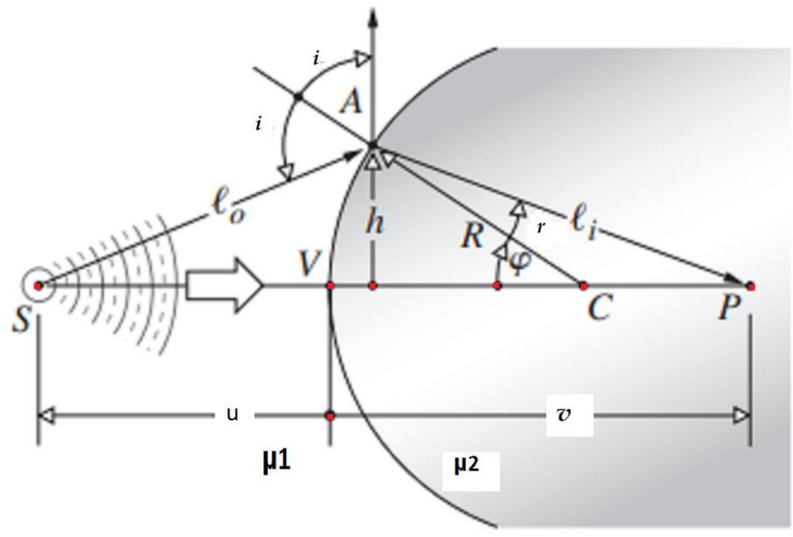


Fig: Refraction at a spherical interface. Conjugate foci.

Using the law of cosines in triangles SAC and ACP along with the fact that $\cos(\varphi) = -\cos(180 - \varphi)$, we get:

$$l_o = \sqrt{R^2 + (u+R)^2 - 2R(u+R)\cos(\varphi)}$$

And

$$l_i = \sqrt{R^2 + (v-R)^2 + 2R(v-R)\cos(\varphi)}$$

The OPL can be rewritten as:

$$OPL = \mu 1\sqrt{R^2 + (u+R)^2 - 2R(u+R)\cos(\varphi)} + \mu 2\sqrt{R^2 + (v-R)^2 + 2R(v-R)\cos(\varphi)}$$

All the quantities in the diagram (u, v, R,etc.) are positive numbers, R constant, and φ position variable thus setting $\frac{dOPL}{d\varphi} = 0$, via Fermat's Principle we have:

$$\frac{\mu 1 R(u+R)\sin(\varphi)}{2l_o} + \frac{\mu 2 R(v-R)\sin(\varphi)}{2l_i} = 0$$

From which it follows that:

$$\frac{\mu 1}{l_o} + \frac{\mu 2}{l_i} = \frac{1}{R}\left[\frac{\mu 2 v}{l_i} - \frac{\mu 1 u}{l_o}\right] \quad \dots (2)$$

Where $u = l_o \cos(\beta)$ and $v = l_i \cos(\beta')$. If we assume small values of φ (i.e., A close to V), $\cos(\beta) \approx 1$ and $\cos(\beta') \approx 1$. Consequently, the expressions for l_i and l_o yield $l_o \approx u,\ l_i \approx v$, and to that approximation:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2 - \mu 1}{R} \quad \ldots (3)$$

If point-F_o or (point-F) in following Fig. is imaged at infinity ($v = \infty$), we have form eq3:

$$\frac{\mu 1}{u} + \frac{\mu 2}{\infty} = \frac{\mu 2 - \mu 1}{R}$$

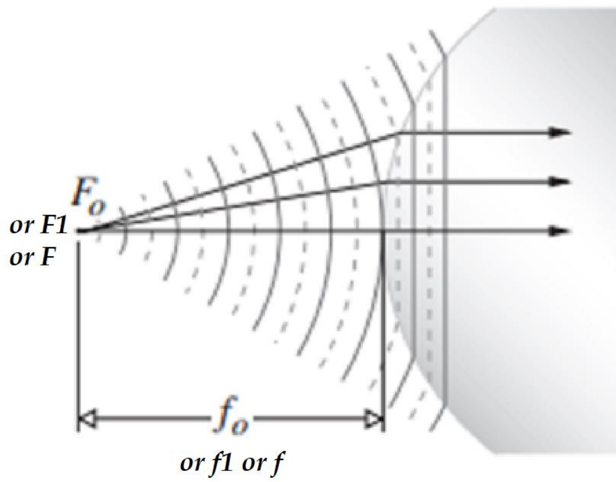


Fig: Plane waves propagating beyond a spherical interface—the object focus.

That special object distance is defined as the **first focal length** or the **object focal length0**, $u = f_1 = f_o = f$ so that:

$$\frac{\mu 1}{f} = \frac{\mu 2 - \mu 1}{R}$$

$$or$$

$$f = \frac{\mu 1}{\mu 2 - \mu 1} R \quad \ldots\ldots (4)$$

Point-F_o or (point-F) is known as the first or object focus. Similarly, the second or image focus is the axial point-F_i or (point-F') where the image is formed when ($u = \infty$); that is:

$$\frac{\mu 1}{\infty} + \frac{\mu 2}{v} = \frac{\mu 2 - \mu 1}{R}$$

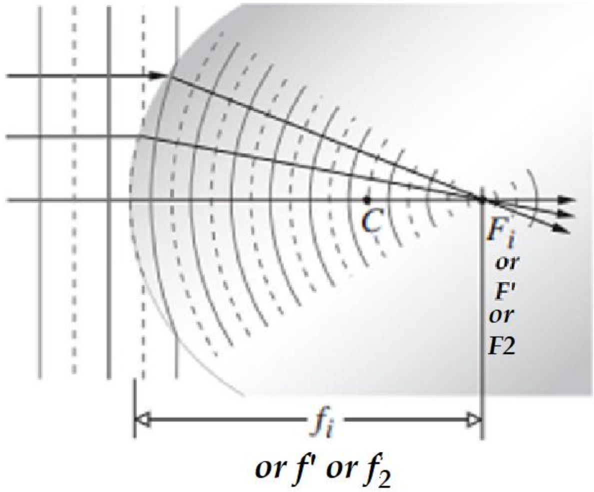


Fig: Plane waves propagating beyond a spherical interface—the object focus.

Defining the second or image focal length f_i or f_2 or f' as equal to v in this special case, we have:

$$f' = \frac{\mu 2}{\mu 2 - \mu 1} R \quad \ldots\ldots(5)$$

- **An object is virtual when the rays converge toward it**, and the object on the right-hand side therefore u will be negative of the vertex, since f_o *or* f *would be negative.*
- **An image is virtual when the rays diverge from it**, and the image on the left-hand side of vertex, therefore v will be a negative quantity.
- The surface is concave, and its radius will also be negative.

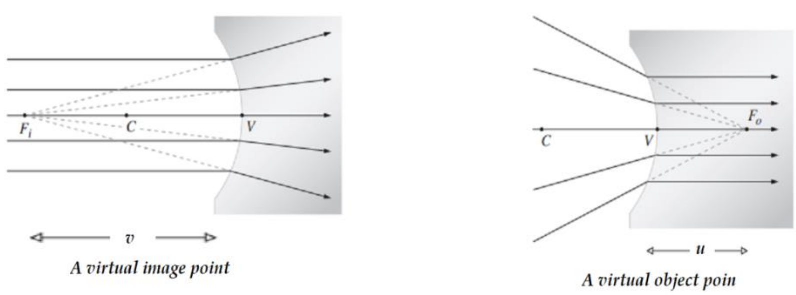


Fig:

Image formation:

The principle of the reversibility of light rays has the consequence that if QM in following Fig. Were an object, an image would be formed at Q′M′. Hence, if any object is placed at the position previously occupied by its image, it will be imaged at the position previously occupied by the object. The object and image are thus interchangeable, or conjugate. Any pair of object and image points such as M an in Fig. are called conjugate points, and planes through these points perpendicular to the axis are called conjugate planes.

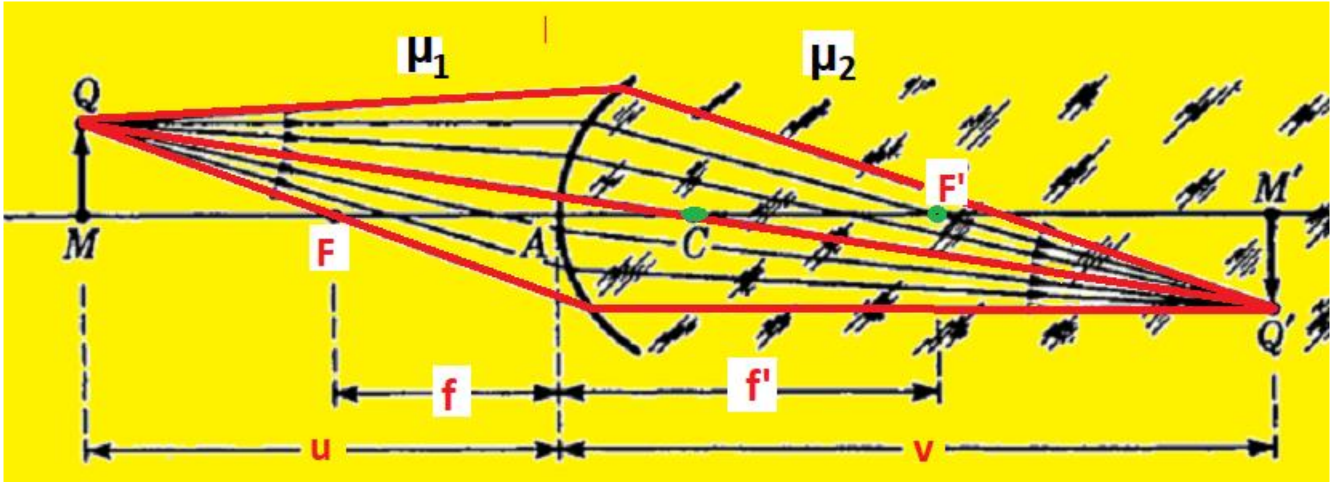


Fig: All rays leaving the object point Q and passing through the refracting surface are brought to a focus at the image point Q'.

As an object is brought closer to the primary focal point F so eq3 become:

$$\frac{\mu 1}{f} + \frac{\mu 2}{\infty} = \frac{\mu 2 - \mu 1}{R}$$

$$\frac{\mu 1}{f} = \frac{\mu 2 - \mu 1}{R} \quad ..(6)$$

Similarly, if the object distance is made larger and eventually approaches infinity, the image distance diminishes and becomes equal to f' in the limit, u = ∞. So eq3 become:

$$\frac{\mu 1}{\infty} + \frac{\mu 2}{f'} = \frac{\mu 2 - \mu 1}{R}$$

$$\frac{\mu 2}{f'} = \frac{\mu 2 - \mu 1}{R} \quad ..(7)$$

Equating the left-hand members of eqs.(6) and (7), we obtain:

$$\frac{\mu 1}{f} = \frac{\mu 2}{f'} = \frac{\mu 2 - \mu 1}{R}$$

$$\therefore \frac{\mu 1}{f} = \frac{\mu 2}{f'}$$

$$\frac{\mu 1}{\mu 2} = \frac{f}{f'} \quad \ldots.(8)$$

So eq3 become:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 1}{f} \quad \text{or} \quad \frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2}{f'} \quad \ldots(9)$$

Problem1: A long horizontal flint-glass ($\mu 2$ = 1.8) cylinder is 20 cm in diameter (D) and has a convex hemispherical left end ground and polished onto it. The device is immersed in ethyl alcohol (μ1 = 1.361) and a tiny LED is located on the central axis in the liquid 80.0 cm to the left of the vertex of the hemisphere. (a) Locate the image of the LED. (b)What would happen if the alcohol was replaced by air?

Sol:

a) Using eq3:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2 - \mu 1}{R}$$

Here $\mu 1$ =1.361, $\mu 2$ = 1.8, u= +80 cm, and R =$\frac{D}{2} = \frac{20}{2} = 10cm$. We can work the problem in centimeters, whereupon the equation becomes:

$$\frac{1.361}{80} + \frac{1.8}{v} = \frac{1.8 - 1.36}{10}$$

$$\therefore v = 66.9cm$$

With the alcohol in place the image is within the glass, 66.9 cm to the right of the vertex ($v > 0$).

b) Removing the liquid, $\mu 1$ =1 for air using eq3 get:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2 - \mu 1}{R}$$

$$\frac{1}{80} + \frac{1.8}{v} = \frac{1.8 - 1}{10}$$

$$\therefore v = 26.7cm$$

The refraction at the interface depends on the ratio ($\mu 2/\mu 1$) of the two indices. The bigger is ($\mu 2 - \mu 1$), the smaller will be v.

Problem2: The end of a solid glass rod of index 1.5 is ground and polished to a hemispherical surface of radius 1cm. A small object is placed in air on the axis 4cm to the left of the vertex. Find the position of the image. Assume $\mu 1 = 1$ for air.

Sol:

The given quantities are $\mu1 = 1$, $\mu2 = 1.5$, $R = 1cm$, and $u = 4cm$. The unknown quantity is v. By direct substitution of the given quantities in eq(1) we obtain:

$$\frac{\mu1}{u} + \frac{\mu2}{v} = \frac{\mu2 - \mu1}{R}$$

$$\frac{1}{4} + \frac{1.5}{v} = \frac{1.5 - 1}{1} = 0.5$$

$$\frac{1.5}{v} = 0.5 - 0.25$$

$$v = \frac{1.5}{0.25} = 6cm$$

One concludes, therefore, that a real image is formed in the glass rod 6cm to the right of the vertex

The following set of sign conventions will be adhered to in our study during the academic year in terms of geometric optics, and it would be good to keep them in mind:

1. All figures are drawn with the **light traveling from left to right**.
2. All object distances **(u)** are considered positive when they are measured to **the left of the vertex** and negative when they are measured to the right.
3. All image distances **(v)** are positive when they are measured to **the right of the vertex** and negative when to the left.
4. Both focal lengths are positive for a converging system and negative for a diverging system.
5. Object and image dimensions **are positive when measured upward** from the axis and **negative when measured downward**.
6. All **convex surfaces are taken as having a positive radius**, and all concave surfaces are taken as having a negative radius.

Problem3: A concave surface with a radius of 4cm separates two media of refractive index $\mu1 = 1$ and $\mu2$=1.5. An object is located in the first medium at a distance of 10cm from the vertex. Find (a) the primary focal length, (b) the secondary focal length, and (c) the image distance.

Sol:

The given quantities are $\mu1 = 1$ and $\mu2 = 1.5$ and $R = -4cm$, and $u = 10cm$. The unknown quantities are f, f', and v.

(a) We use eq6 directly to obtain:

$$\frac{\mu 1}{f} = \frac{\mu 2 - \mu 1}{R}$$

$$\frac{1}{f} = \frac{1.5 - 1}{-4}$$

$$f = \frac{-4}{0.5} = -8cm$$

(a) We use eq7 directly to obtain:

$$\frac{\mu 2}{f'} = \frac{\mu 2 - \mu 1}{R}$$

$$\frac{1.5}{f'} = \frac{1.5 - 1}{-4}$$

$$f' = \frac{-6}{0.5} = -12cm$$

Note that in this problem both focal lengths are negative and that the ratio f/f' is 1/1.5 as required by eq9. The minus signs indicate a diverging system. (c) We use eq9 and obtain, by direct substitution,

$$\frac{1}{10} + \frac{1.5}{v} = -\frac{1}{8}$$

$$v = -6.66cm$$

The image is located 6.66 cm from the vertex A, and the minus sign shows it is to the left of A and therefore virtual, as shown in following Fig:

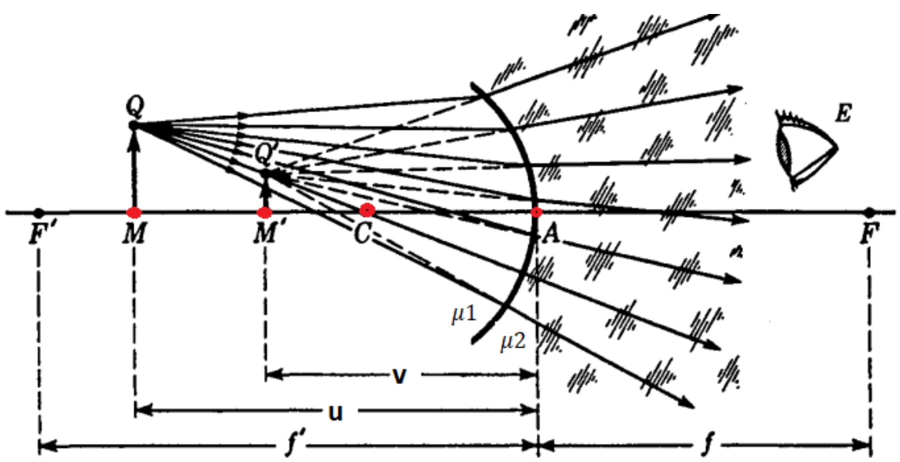


Fig: All rays leaving the object point Q, and passing through the refracting surface appear to be coming from the virtual image point Q'.

Magnification:

In any optical system the ratio between the transverse dimension of the final image and the corresponding dimension of the original object is called the lateral magnification. To determine the relative size of the image formed by a single spherical surface, reference is made to the geometry of following fig. Here the undeviated ray 5 forms two similar right triangles QMC and Q'M'C. The theorem of the proportionality of corresponding sides requires that:

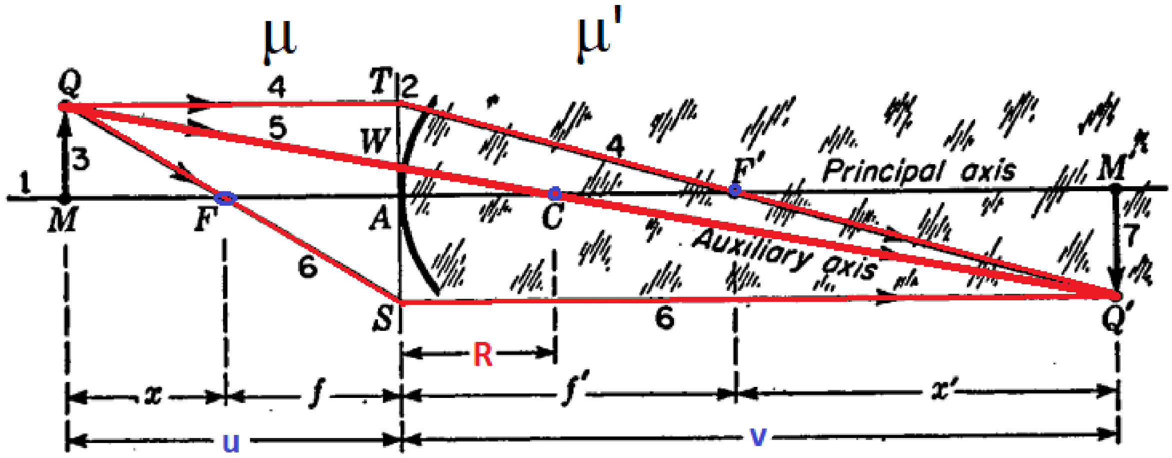


Fig: image formed by a single sphericalsurface.

The theorem of the proportionality of corresponding sides requires that:

$$\frac{M'Q'}{MQ} = \frac{CM'}{CM} \quad or \quad \frac{-y'}{y} = \frac{v-R}{u-R}$$

We now define y' /y as the lateral magnification m and obtain:

$$m = \frac{-y'}{y} = -\frac{v-R}{u-R} \quad \ldots (10)$$

If m is positive, the image will be virtual and erect, while if it is negative, the image is real and inverted.

Here we will introduce new quantities to represent eq 9:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 1}{f} \quad \text{or} \quad \frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2}{f'}$$

$$P = \frac{\mu 1}{f} \;, \quad P' = \frac{\mu 2}{f'} \quad \ldots (11)$$

Where the quantities (P and P′) are, the refracting power.

Note: ***when(f and f*') are measured in meters(m), the power (P and P') are in units called diopters(D).***

So eq9 can be rewrite as follow:

$$P = \frac{\mu 2 - \mu 1}{R} \quad ..(12)$$

$$P = \frac{\mu 1}{u} + \frac{\mu 2}{v} \quad ..(13)$$

Problem4: One end of a glass rod of refractive index 1.50 is ground and polished with a convex spherical surface of radius 10 cm. An object is placed in the air on the axis 40 cm to the left of the vertex. Find (a) the power of the surface and (b) the position of the image.

Sol:

The given quantities are $\mu 1 = 1$, $\mu 2 = 1.5$, $R = 10cm = 0.1m$, and $u = 40cm = 0.4m$. The unknown quantities are P and v. To find the solution to (a), we make use of eq12, substitute the given distance in meters, and obtain

$$P = \frac{\mu 2 - \mu 1}{R} = \frac{1.5 - 1}{0.1} = 5D$$

Now using eq13 get:

$$P = \frac{\mu 1}{u} + \frac{\mu 2}{v}$$

$$5 = \frac{1}{0.4} + \frac{1.5}{v}$$

$$5 - 2.5 = \frac{1.5}{v}$$

$$v = \frac{1.5}{2.5} = 0.6\text{m} = 60\text{cm}$$

Thin-Lens Equations:

Return to the discussion of refraction at a single spherical interface, where the location of the conjugate points-S and -P is given by:

$$\frac{\mu 1}{u} + \frac{\mu 2}{v} = \frac{\mu 2 - \mu 1}{R}$$

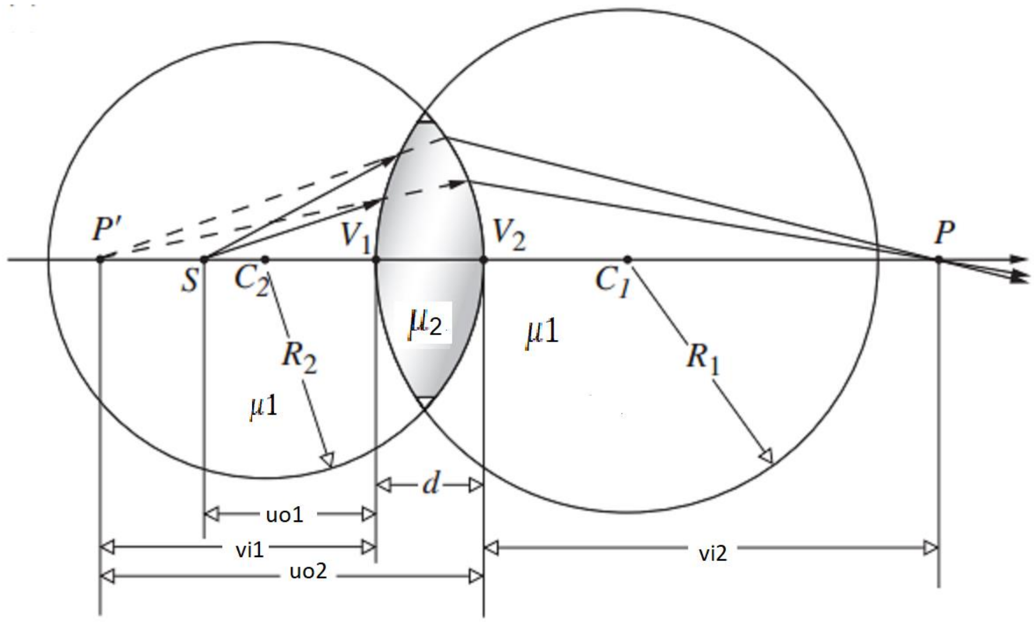


Fig: A spherical lens. (a) Rays in a vertical plane passing through a lens. Conjugate foci. (b) Refraction at the interfaces where the lens is immersed in air and $\mu 1 < \mu 2$. The radius drawn from C1 is normal to the first surface, and as the ray enters the lens it bends down toward that normal. The radius from C2 is normal to the second surface; and as the ray emerges, since nl 7 na, the ray bends down away from that normal. (c) The geometry

For double spherical surfaces shown in above fig., can be writing the following equation:

$$\frac{\mu 1}{uo1} + \frac{\mu 1}{vi2} = (\mu 2 - \mu 1)\left(\frac{1}{R1} - \frac{1}{R2}\right) + \frac{\mu 2 d}{(vi1 - d)vi1} \quad \dots (14)$$

If the lens is thin enough (d → 0), the last term on the right is effectively zero. As a further simplification, assume the surrounding medium to be air (i.e., n1 ≈ 1). Accordingly, we have the very useful Thin-Lens Equation, often referred to as the Lensmaker's Formula:

$$\frac{1}{uo1} + \frac{1}{vi2} = (\mu 2 - 1)\left(\frac{1}{R1} - \frac{1}{R2}\right) \quad \dots or$$

$$\frac{1}{u} + \frac{1}{v} = (\mu 2 - 1)\left(\frac{1}{R1} - \frac{1}{R2}\right) \quad \dots (15)$$

Where $u = uo1 \; and \; v = vi2.$ The points-V1 and -V2 tend to coalesce as d→0, so that u and v can be measured from either the vertices or the lens center.

Just as in the case of the single spherical surface, if u is moved out to infinity, the image distance becomes the focal length $f' or \; f_i$, or symbolically:

$$\lim_{u \to \infty} v = f' = f_i$$

Similarly

$$\lim_{v \to \infty} u = f = f_o$$

It is evident from eq15 that for a thin lens $f' = f$, and consequently we drop the subscripts altogether. Thus

$$\frac{1}{f} = (\mu 2 - 1)\left(\frac{1}{R1} - \frac{1}{R2}\right) \dots (16)$$

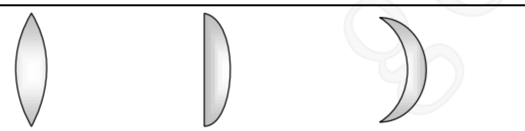
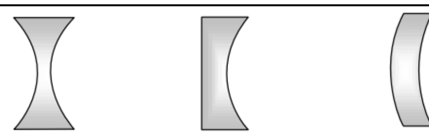
And

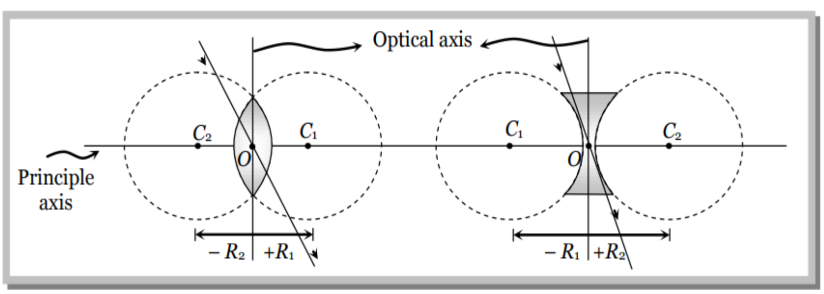
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \dots (17)$$

Notes and Some definitions

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) **Type of lenses**

Convex lens (Converges the light rays)	**Concave lens (Diverges the light rays)**
	
Double convex Plano convex Concavo convex	Double concave Plane concave Convexo concave
Thick at middle	Thin at middle
It forms real and virtual images both	It forms only virtual images



C_1, C_2 – Centre of curvature,
R_1, R_2 – Radii of curvature

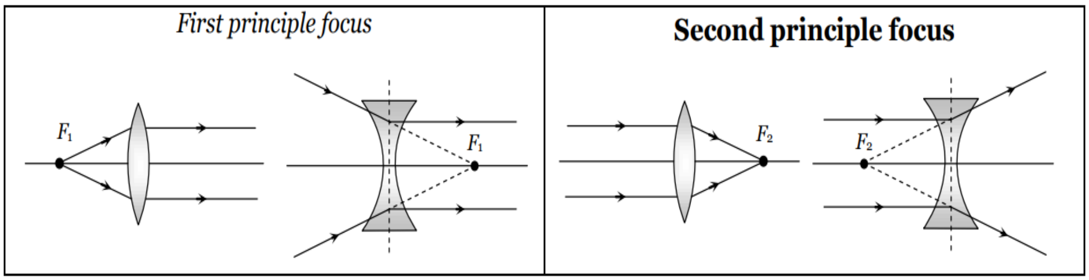
(i) Lens maker's formula The relation between *f*, *μ*, *R1* and *R2* is known as lens maker's formula and it is:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Equiconvex lens	Plano convex lens	Equi concave lens	Plano concave lens
$R_1 = R$ and $R_2 = -R$ $f = \frac{R}{2(\mu-1)}$ for $\mu = 1.5$, $f = R$	$R_1 = \infty, R_2 = -R$ $f = \frac{R}{(\mu-1)}$ for $\mu = 1.5$, $f = 2R$	$R_1 = -R$, $R_2 = +R$ $f = -\frac{R}{2(\mu-1)}$ for $\mu = 1.5$ $f = -R$	$R_1 = \infty$, $R_2 = R$ $f = \frac{R}{2(\mu-1)}$ for $\mu = 1.5$, $f = -2R$

(ii) Optical centre (O) : A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical centre).

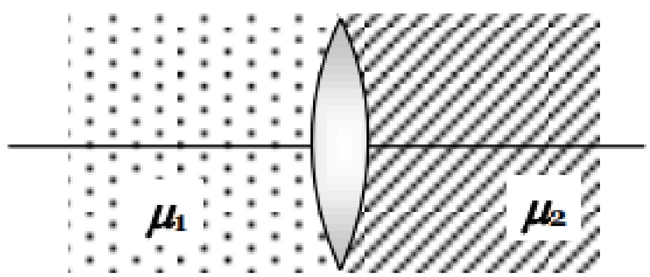
(iii) Principle focus



(iv) Second principle focus is the principle focus of the lens.

- When medium on two sides of lens is same then |F1|= |F2 |.
- If medium on two sides of lens are not same then the ratio of two focal lengths

$$\frac{f}{f'} = \frac{f_1}{f_2} = \frac{\mu 1}{\mu 2}$$



iv) Focal length (f) : Distance of second principle focus from optical center is called focal length:

$$f_{convex} = positive$$

$$f_{concave} = negative$$

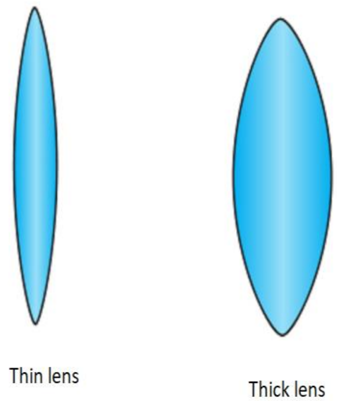
$$f_{plane} = \infty$$

(v) Power of lens (P) : Means the ability of a lens to converge the light rays. Unit of power is Diopter (D).

$$P = \frac{1}{f} \quad must\ be\ used\ f\ in\ meter(m)\ unit\ to\ get\ P\ in\ diopter(D) unit$$

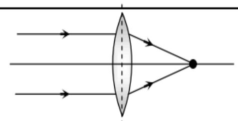
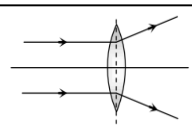
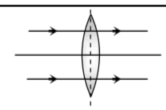
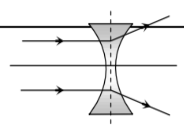
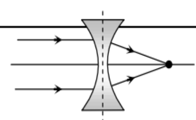
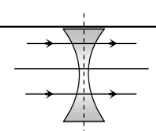
$$P_{convex} = positive, \quad P_{concave} = negative \quad and \quad P_{plane} = zero$$

(vi) Thin lens: $P \downarrow f \uparrow R \uparrow$, while for thick lens: $P \uparrow f \downarrow R \downarrow$



(vii) Focal length of a glass lens (μ = 1.5) is f in air then inside the water it's focal length is 4f. In liquids focal length of lens increases (↑) and it's power decreases (↓).

(viii) Opposite behavior of a lens: In general refractive index of lens (μ_L) > refractive index of medium surrounding it (μ_M).

$\mu_L > \mu_M$	$\mu_L < \mu_M$	$\mu_L = \mu_M$
		
		

Magnification by lens:

A simple formula for the image magnification produced by a single lens can be derived from the geometry of following fig: In accord with convention, transverse distances above the optical axis are taken as positive quantities, and those below the axis are given negative numerical values. Therefore $y_o>0$ and $y_i<0$. Observe that triangles AOF_i and $P_2P_1F_i$ are similar. Ergo:

$$\frac{\boldsymbol{y_o}}{|\boldsymbol{y_i}|} = \frac{\boldsymbol{f}}{\boldsymbol{v} - \boldsymbol{f}}$$

In the same way, triangles S2S1O and P2P1O are similar, and:

$$\frac{\boldsymbol{y_o}}{|\boldsymbol{y_i}|} = \frac{\boldsymbol{u}}{\boldsymbol{v}}$$

The ratio of the transverse dimensions of the final image formed by any optical system to the corresponding dimension of the object is defined as the lateral or transverse magnification, M, that is:

$$M = \frac{y_i}{y_o} \quad \ldots (18)$$

$$M = -\frac{v}{u} \quad (19)$$

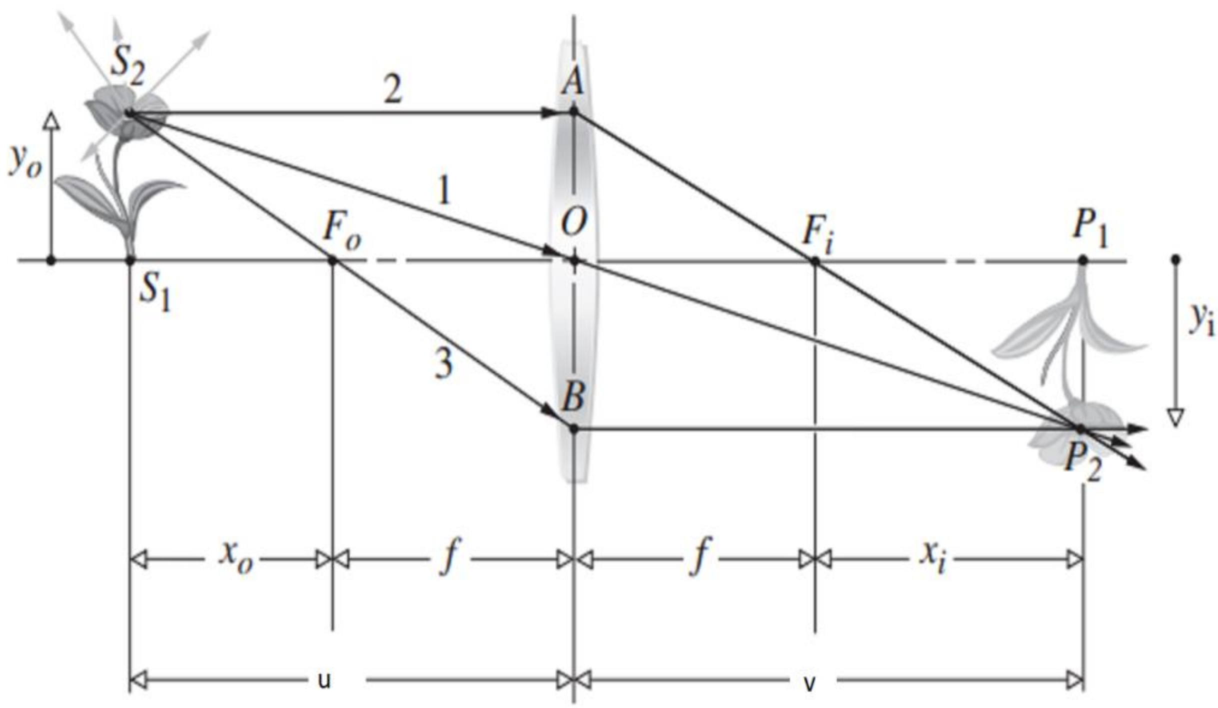


Fig: Object and image location for a thin lens.

A **positive M connotes an erect image, while a negative value means the image is inverted**. Bear in mind that $\boldsymbol{v}$ and u are both positive for real objects and images.

TABLE Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters		
Quantity	Sign	
	+	−
u	Real object	Virtual object
v	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect object	Inverted object
y_i	Erect image	Inverted image
M	Erect image	Inverted image

Problem5: A biconvex (also called a double convex) thin spherical lens has radii of 100 cm and 20 cm. The lens is made of glass with an index of 1.54 and is immersed in air. (a) If an object is placed 70 cm in front of the 100-cm surface, locate the resulting image and describe it in detail. (b) Determine the transverse magnification of the image. (c) Draw a ray diagram

Sol:

(a) We don't have the focal length, but we do know all the physical parameters, so eq16) comes to mind:

$$\frac{1}{f} = (\mu 2 - 1)(\frac{1}{R1} - \frac{1}{R2})$$

$$\frac{1}{f} = (1.54 - 1)\left(\frac{1}{100} - \frac{1}{-20}\right) = 0.54 \times \frac{6}{100}$$

$$f = 30.86cm$$

Now we can find the image. Since $= 70cm$, that's greater than $2f$—hence, even before we calculate v,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{70} + \frac{1}{v} = \frac{1}{30.86}$$

$$v = \ 55.19 \text{ cm}$$

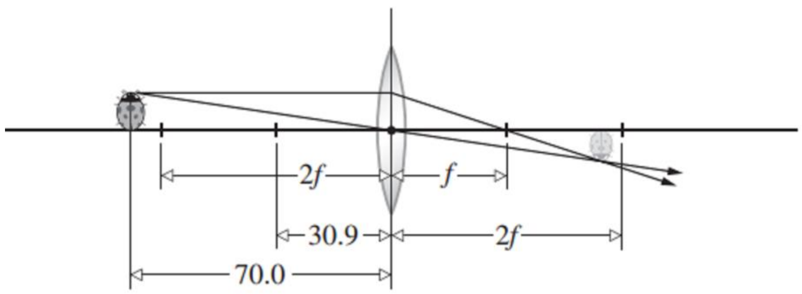
The image is real, inverted between f and $2f$ on the right of the lens. Note that $v > 0$, which means the image is real.

(b) The magnification follows from:

$$\boldsymbol{M = -\frac{v}{u} = -\frac{55.19}{70} = -0.788}$$

and the image is inverted (M<0) and minified ($|M| > 1$).

(c) Draw the lens and mark out two focal lengths



Problem6: Both surfaces of an equi-convex thin spherical lens have the same curvature. A 2-cm-tall bug is on the central axis 100 cm from the front face of the lens. The image of the bug formed on a wall is 4 cm tall. Given that the glass of the lens has an index of 1.5, find the radii of curvature of the surfaces:

Sol:

We have y_o = 2 cm, $u = 100cm$, R1 = R2, |yi| $= 4cm$, and $\mu 2 = 1.5$. We also know that the image is real, so it must be inverted and therefore y_i = -4cm —that's crucial! To find the radii we'll need eq18and eq19. and the focal length. We can compute f if we first determine v. Hence, knowing M:

$$\boldsymbol{M = \frac{y_i}{y_o} = \frac{-4}{2} = -2}$$

$$\boldsymbol{M = -\frac{v}{u}}$$

$$\boldsymbol{v = -M \times u = -(-2) \times 100 = 200cm}$$

Using the Gaussian Lens formula eq17:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{100} + \frac{1}{200}$$

$$f = \frac{200}{3} = 66.67cm$$

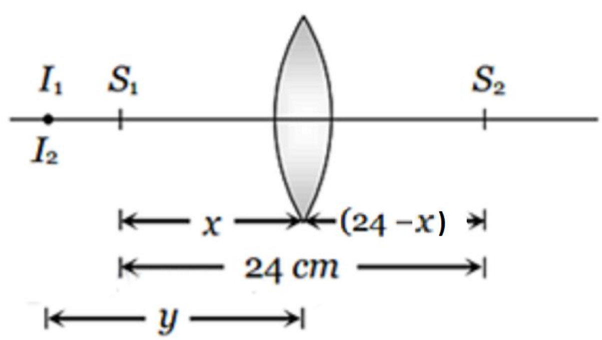
The Lens maker's Formula will give us R:

$$\frac{1}{f} = (\mu 2 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{2}\frac{2}{R} = \frac{1}{R}$$

$$\therefore R = f = 66.67cm$$

Problem7: Two point light sources are 24 cm apart. Where should a convex lens of focal length 9 cm be put in between them from one source so that the images of both the sources are formed at the same place : (a) 6 cm (b) 9 cm (c) 12 cm (d) 15 cm

Sol:
Solution (a)
The given condition will be satisfied only if one source (S1) placed on one side such that u < f (i.e. it lies under the focus). The other source (S2) is placed on the other side of the lens such that u > f (i.e. it lies beyond the focus).



If S1 is object for the lens then: $\frac{1}{f} = \frac{1}{u1} + \frac{1}{v1} = \frac{1}{x} + \frac{1}{-y}$ image on left –ve, object on right +ve

$$\frac{1}{y} = \frac{1}{x} - \frac{1}{f} \quad \ldots (i)$$

If S2 is object for the lens then: $\frac{1}{f} = \frac{1}{u2} + \frac{1}{v2}$ image on left –ve, object on right -ve

$$\frac{1}{-f} = \frac{1}{-(24-x)} + \frac{1}{-y}$$

$$\frac{1}{y} = \frac{1}{f} - \frac{1}{(24-x)} \quad ..(ii)$$

Form eq(i) and eq(ii):

$$\frac{1}{y} = \frac{1}{x} - \frac{1}{f} = \frac{1}{f} - \frac{1}{(24-x)}$$

$$\frac{1}{x} - \frac{1}{f} = \frac{1}{f} - \frac{1}{(24-x)}$$

$$\frac{1}{x} + \frac{1}{(24-x)} = \frac{2}{f}$$

$$\frac{(24-x)+x}{24x-x^2} = \frac{2}{9}$$

$$216 = 48x - 2x^2$$

$$2x^2 - 48x + 216 = 0$$

$$x^2 - 24x + 108 = 0$$

you can use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4 \times 1 \times 108}}{2 \times 1}$$

$$x = \frac{24 \pm \sqrt{144}}{2} = \frac{24 \pm 12}{2} = 12 \pm 6$$

$$\therefore x1 = 18 \; and \; x2 = 6$$

References

1. Eugene Hecht ,"Optics "fifth edition, © Pearson Education Limited 2017,
2. Francis A. Jenkins , and Harvey E.White, " Fundamental of optics" Fourth Edition, McGraw-Hill Higher Education 1981.
3. N. Subrahmanyam, and Brij Lal ,"A textbook of optics ", S. CHAND & Company LTD. (AN ISO 9001: 2000 company) RAM NAGAR, New Delhui-110 055, 2000.