الجامعة المستنصرية /كلية العلوم

قسم الفيزياء

Mustansiriyah University

College of science

Physics department

Optics

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Lecture (9)

For 3rd year Students

Lecture Title: Plane and Spherical mirrors

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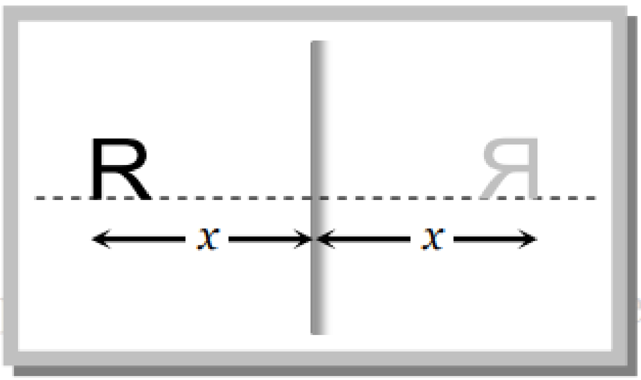
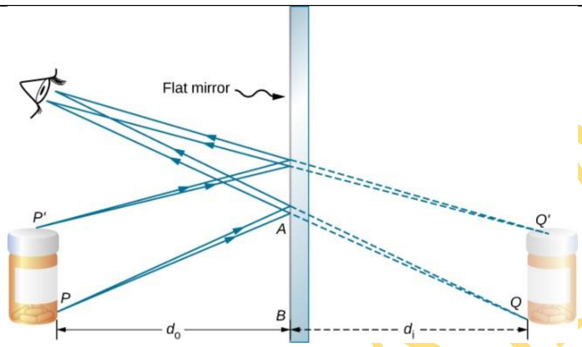
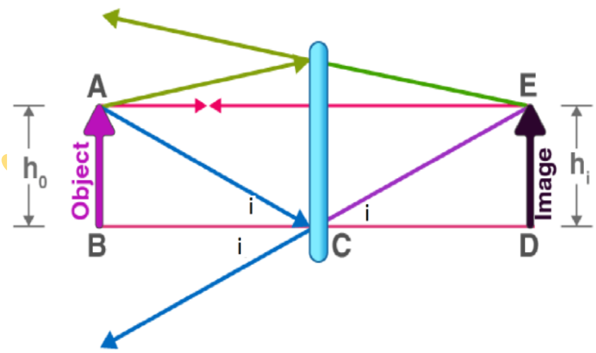
Plane Mirror

The image formed by a plane mirror is virtual, erect, laterally inverted, equal in size that of the object and at a distance equal to the distance of the object in front of the mirror.







A plane mirror is a smooth or highly polished plane surface that reflects light **in order to form an image. It is a flat, polished, and reflective surface that produces a** virtual image **of the real object.**

Characteristics of a plane mirror: The characteristics of the plane mirror are:

- Images formed are erect and of the same size as the object.
- The distance between the image and the distance of the objects is equal to the plane mirror.
- It forms laterally inverted images, meaning that images are not reversed from top to bottom, but are reversed from left to right.
- Image formation by a plane mirror is a virtual image.
- The radius of curvature of a plane mirror is infinite.

1. What do you mean by a plane mirror?

A flat, polished, and reflective surface that produces a virtual erect image of the real object is called a plane mirror.

2. What is the image formed by a plane mirror?

The images formed by a plane mirror are virtual, erect, and of the same size as an object.

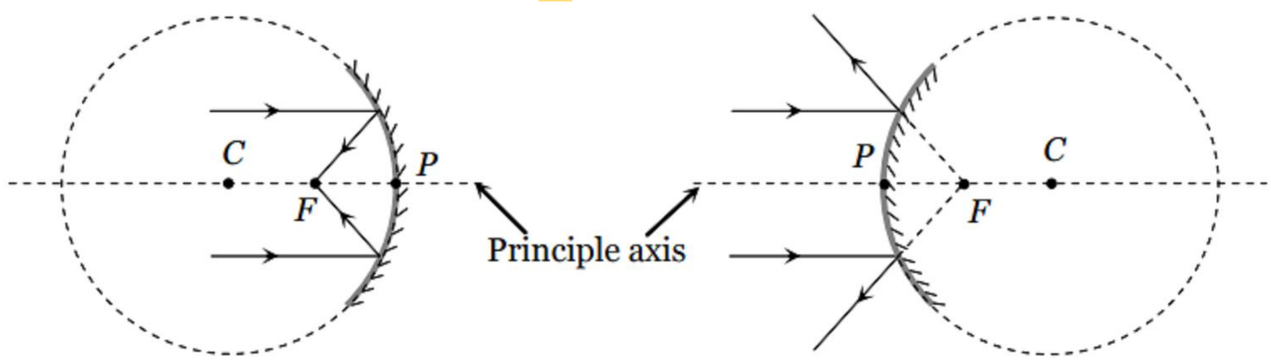
3. What are the characteristics of a plane mirror?

The characteristics of a plane mirror are:

- Formed images are virtual
- It forms an erect image
- The distance between the image and object is equal to the plane mirror.
- It exclusively forms laterally inverted images.

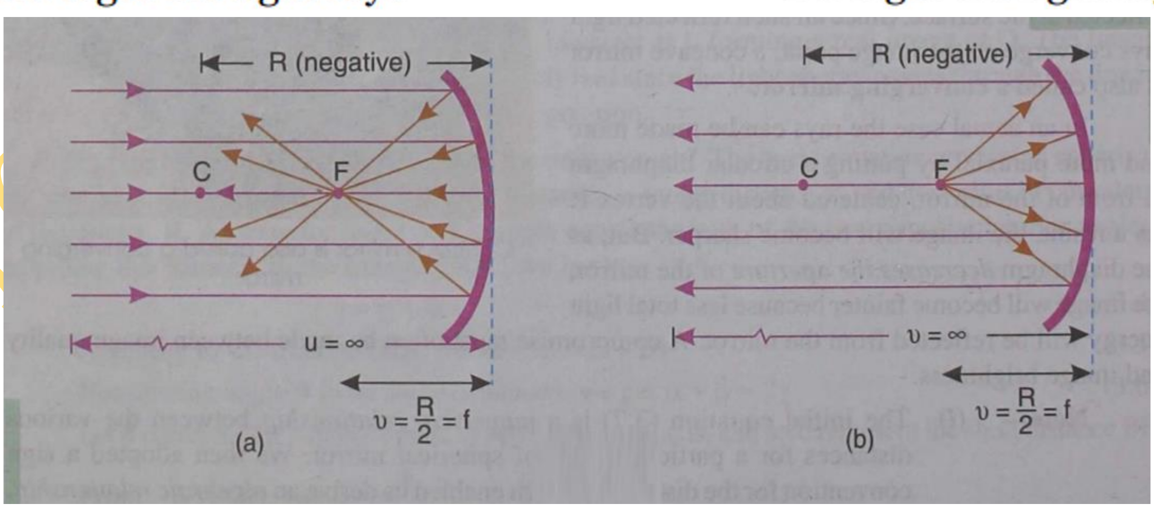
The spherical Mirrors

Curved mirrors that form images very much like those of lenses or curved refracting surfaces have been known since the time of the ancient Greeks. It is a part of a transparent hollow sphere whose one surface is polished:

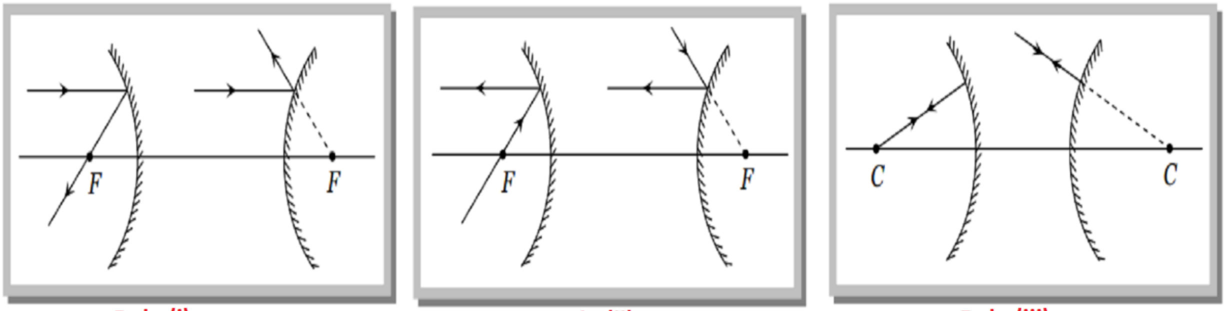


Converges the light rays

Diverges the light rays



Rules of image formation and sign convention:



Rule (i) Rule (ii) Rule (iii)

The spherical Mirror Formula:

The paraxial equation that relates conjugate object and image points to the physical parameters of a spherical mirror can be derived with the help of following Fig.

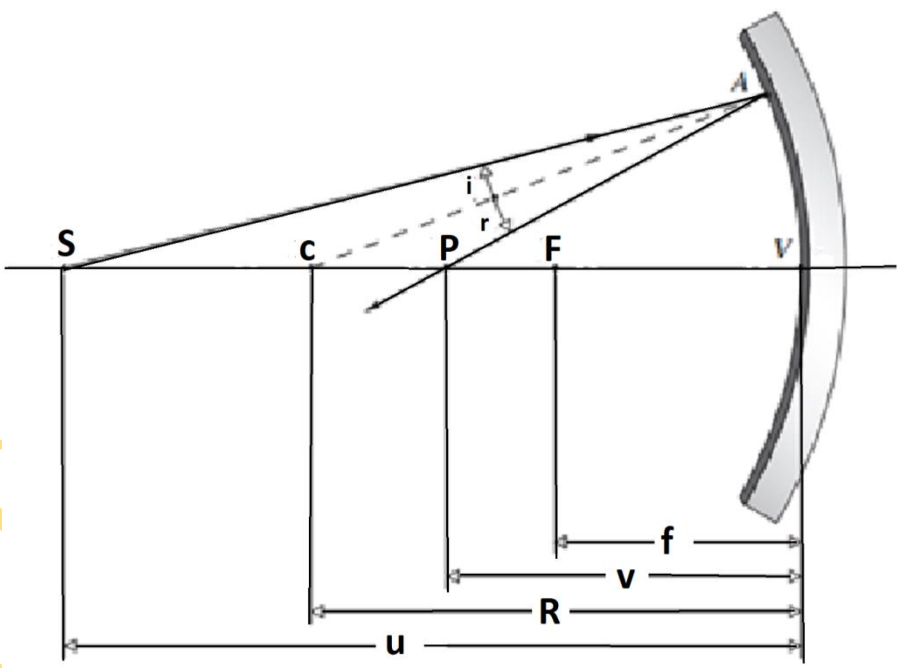


Fig: A concave spherical mirror. Conjugate foci.

To that end, observe that since i=r, the angle ∠SAP is bisected by CA, which therefore divides the side SP of triangle SAP into segments proportional to the remaining two sides; that is:

$$\frac{\overline{SC}}{\overline{SA}} = \frac{\overline{CP}}{\overline{PA}} \quad \ldots (1)$$

Furthermore: $\overline{SC} = u - |R| and \quad \overline{CP} = |R| - v$

where u and v are on the left and therefore positive. Using the same sign convention as we did with refraction, R will be negative because C is to the left of V (i.e., the surface is concave). Thus $|R| = -R$ and:

$$\overline{SC} = u + R \qquad and \qquad \overline{CP} = -(v + R)$$

In the paraxial region $\overline{SA} \approx u, \overline{PA} \approx v$, and eq1 becomes:

$$\frac{u+R}{u} = -\frac{v+R}{v} \qquad \text{or}$$

$$\boldsymbol{\frac{1}{u} + \frac{1}{v} = -\frac{2}{R}} \qquad \dots (2)$$ This is the Mirror Formula.

It's equally applicable to concave (R<0) and convex (R>0) mirrors.

The primary or object focus is again defined by:

$$\lim_{v\to\infty} u = f \qquad ..(3)$$

Consequently, from eq2 get:

$$\frac{1}{f} + \frac{1}{\infty} = -\frac{2}{R} \qquad \text{So} \qquad \boldsymbol{f = -\frac{R}{2}}$$

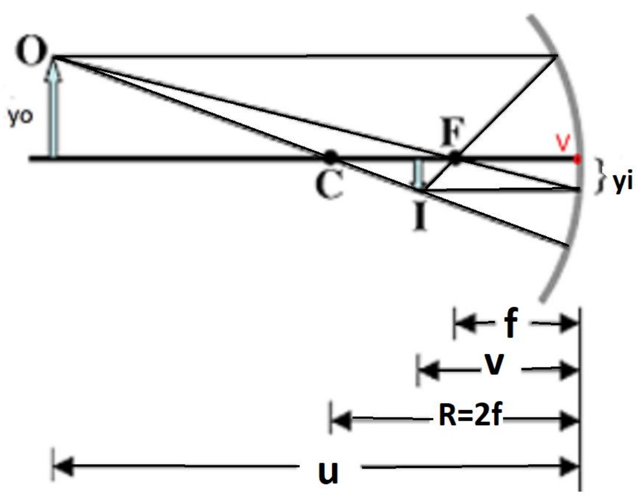
Dropping the subscripts on the focal lengths yields:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad \dots (4)$$

Observe that f will be positive for concave mirrors (R < 0) and negative for convex mirrors (R>0).

Magnification for spherical mirror see the following fig:

$$\boldsymbol{M = \frac{yi}{yo} = -\frac{v}{u} = -\frac{f}{u}} \qquad \dots (5)$$



Important Notes:

1. Distances measured from **left to right are positive** while those measured **from right to left are negative**.
2. Incident rays travel from left to right and reflected rays from right to left.
3. The focal length is measured from the focal point to the vertex. This gives **f a positive sign for concave** mirrors and **a negative sign for convex mirrors**.
4. The radius **R** is measured from the vertex to the center of curvature. This makes **R negative for concave mirrors and positive for convex mirrors.**
5. Object distances **u** and image distances **v** are measured from the object and from the image respectively to the vertex. This makes both **u and v positive** and the object and image real when they **lie to the left of the vertex;** they are negative and virtual when they lie to the right

TABLE Sign Convention for Spherical Mirrors

Quantity	Sign +	Sign −
u	Left of V, real object	Right of V, virtual object
v	Left of V, real image	Right of V, virtual image
f	Concave mirror	Convex mirror
R	C right of V, convex	C left of V, concave
y_o	Above axis, erect object	Below axis, inverted object
y_i	Above axis, erect image	Below axis, inverted image

Problem1: A small frog is sitting on the central axis 35cm in front of a concave spherical mirror having a focal length of 20cm. Locate the image and describe it completely. What is the transverse magnification of the image?

Sol:

Using eq9 get:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{35}+\frac{1}{v}=\frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{35} = 0.021\,43$$

$$v = 46.67cm$$

The image is real, inverted, and magnified. Notice that si is positive, which means the image is real.

$$M = -\frac{v}{u} = -\frac{46.67}{35} = -1.3$$

The minus sign means the image is inverted.

Problem2: An object 2cm high is situated 10cm in front of a concave mirror of radius 16cm. Find (a) the focal length of the mirror, (b) the position of the image, and (c) the lateral magnification.

Sol

The given quantities are yo= 2cm, u = + 10cm, and R = -16cm. The unknown quantities are **f, v, and m.**

$$f = -\frac{R}{2} = -\frac{-16}{2} = 8cm$$

Use eq4 get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{10} + \frac{1}{v} = \frac{1}{8}$$

$$\frac{1}{v} = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$$

$$v = 40cm$$

$$M = -\frac{v}{u} = -\frac{40}{10} = -4$$

The image occurs 40cm to the left of the mirror, is 4times the size of the object, and is real and inverted.

Problem3: A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole find the distance of produces an image .

Sol: u=-f;

Using:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{-f}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{2}{f}$$

$$v=\frac{f}{2}$$

Problem4: A point light source lies on the principal axis of a concave spherical mirror with radius of curvature 160cm. Its vertical image appears to be at a distance of 70cm from it. Determine the location of the light source

Sol:

Here v = 70 cm, R = -160 cm

Using:

$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}$$

$$\frac{1}{u}+\frac{1}{70}=\frac{2}{160}$$

$$u=-555.56cm=-5.56m$$

The light is at a distance of 5.56m in front (to the left of the vertex) of the mirror.

Problem5: A point source of light is located 20cm in front of a convex mirror with f =15cm. Determine the position and character of the image point:

Sol

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{20}+\frac{1}{v}=\frac{1}{-15}$$

$$v=-8.57cm$$

As v is positive, the image is located behind (to the right side of the vertex of) the mirror. Hence, the image is virtual.

Table **Physical Significance of the Signs of Spherical Mirror Parameters**

Quantity	Sign	
	+	−
u	real object	virtual object
v	real image	virtual image
f	concave mirror	convex mirror
y_o	erect object	inverted object
y_i	erect image	inverted image
M_T	erect image	inverted image
R	convex mirror	concave mirror

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=-\frac{2}{R}$$ The mirror equation

Problem6: Show that the spherical mirror equation is applicable to the plane reflecting surface.

Sol

For the planar surface the radius of the curvature becomes infinite, hence

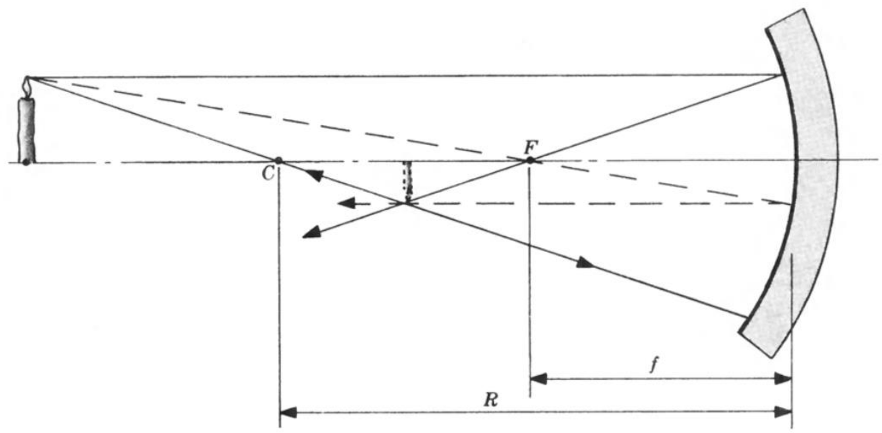
$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}=-\frac{2}{\infty}=0$$

$$\frac{1}{u}=-\frac{1}{v}$$

Or $u=-v$, as required. Since the object distance is positive, $v\ must\ be\ a\ negative\ number$; the image is to the right of the interface.

Problem7: a concave mirror has a radius of magnitude |R| and is centered at C. Areal erect object $\frac{|R|}{6}$ tall is located a distance 1.5|R| from the mirror's vertex. Draw arsy diagram showing the formation of the image. Then compute the magnification and image location.

Sol



Because R is –ve can be write R=-|R|. Used

$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}\ become$$

$$\frac{1}{1.5|R|}+\frac{1}{v}=-\frac{2}{-|R|}=\frac{2}{|R|}$$

Or

$v=\frac{3|R|}{4}$ The image is real and to the left of vertex

The magnification given by:

$M=-\frac{v}{u}=-\frac{\frac{3|R|}{4}}{1.5|R|}=-0.5,$ the image is inverted and half sized.

Problem8: The one inch tall candle is set three inches in front of a concave mirror having a one-foot radius. Describe the resulting image:

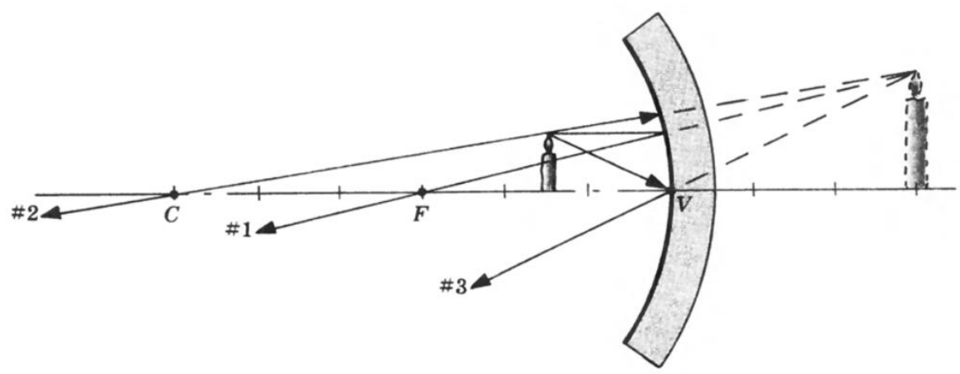
Sol

$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}$$

$$\frac{1}{3}+\frac{1}{v}=-\frac{2}{-12}$$

$v=-6inches$ The image is virtual.

$M=-\frac{v}{u}=-\frac{-6}{2}=+2,$ the image is erected and twice the size of the object.



Problem9: A concave mirror of 20cm radius is to be used to project the image of candle onto a wall 110cm away. Where will the candle have to be placed and will the image look like?

Sol:

The object distance is to the left of V if the image is to be real, hence $v=110cm\ and$

$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{R}$$

$$\frac{1}{u}+\frac{1}{110}=-\frac{2}{-20}$$

$$u=+11cm.$$

$M=-\frac{v}{u}=-\frac{110}{11}=-10,$ is the real inverted image and magnified 10 times.

Problem10: Design a spherical mirror which will from the erect half-sized image of the object if that object is 100cm from the vertex. Where will the image be located?

Sol

We can determine the image distance from the magnification as follows:

$$M = -\frac{v}{u} = \frac{1}{2}$$

$$-\frac{v}{100} = \frac{1}{2}$$

Hence

$$v = -50cm$$

The mirror equation now yields the relation:

$$\frac{1}{u} + \frac{1}{v} = -\frac{2}{R}$$

$$\frac{1}{100} + \frac{1}{-50} = -\frac{2}{R}$$

$$R = +200cm$$

The mirror is convex (only convex mirror generate an erect minified image)

References

1. Eugene Hecht ,"Optics "fifth edition, © Pearson Education Limited 2017,
2. Francis A. Jenkins , and Harvey E.White, " Fundamental of optics" Fourth Edition, McGraw-Hill Higher Education 1981.
3. N. Subrahmanyam, and Brij Lal ,"A textbook of optics ", S. CHAND & Company LTD. (AN ISO 9001: 2000 company) RAM NAGAR, New Delhui-110 055, 2000.