الجامعة المستنصرية /كلية العلوم

قسم الفيزياء

Mustansiriyah University

College of science

Physics department

Optics

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Lecture (7.1)

For 3rd year Students

Lecture Title: Notes and equations on Spherical Surfaces and Lenses with Solved Problems

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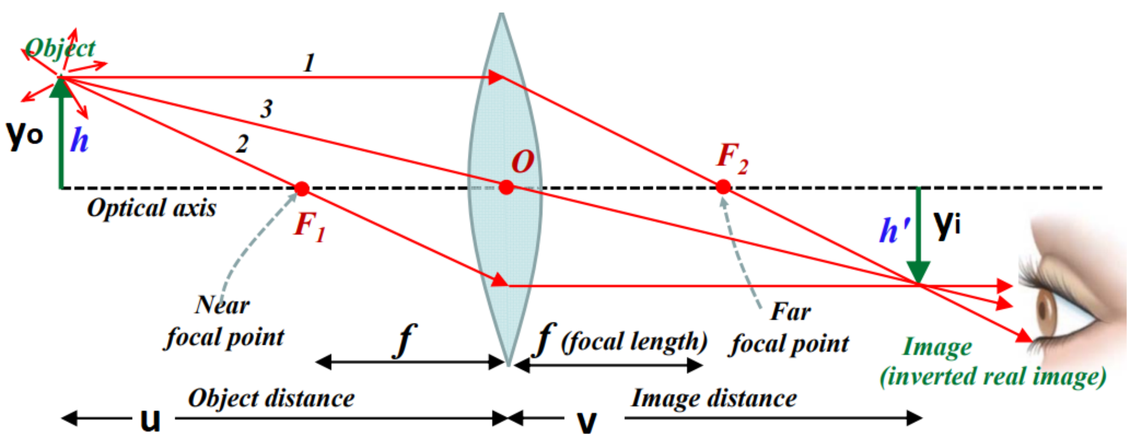
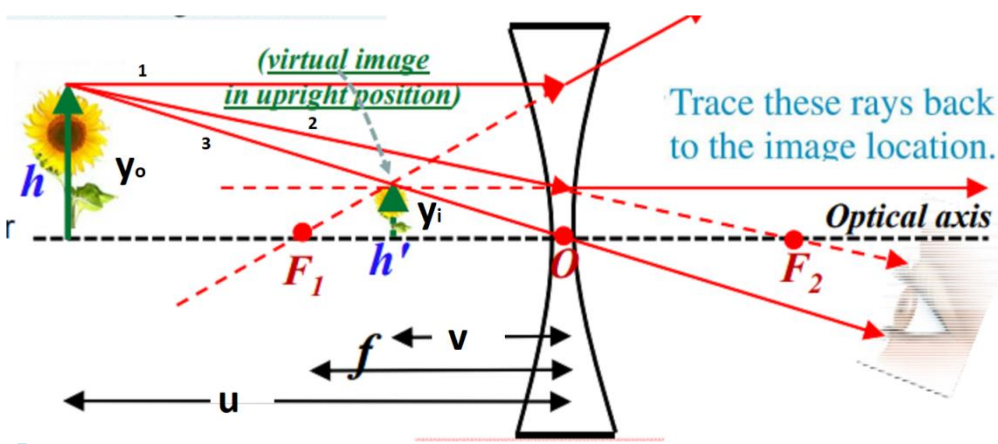
Notes and Laws of Spherical Surface and thin lenses:

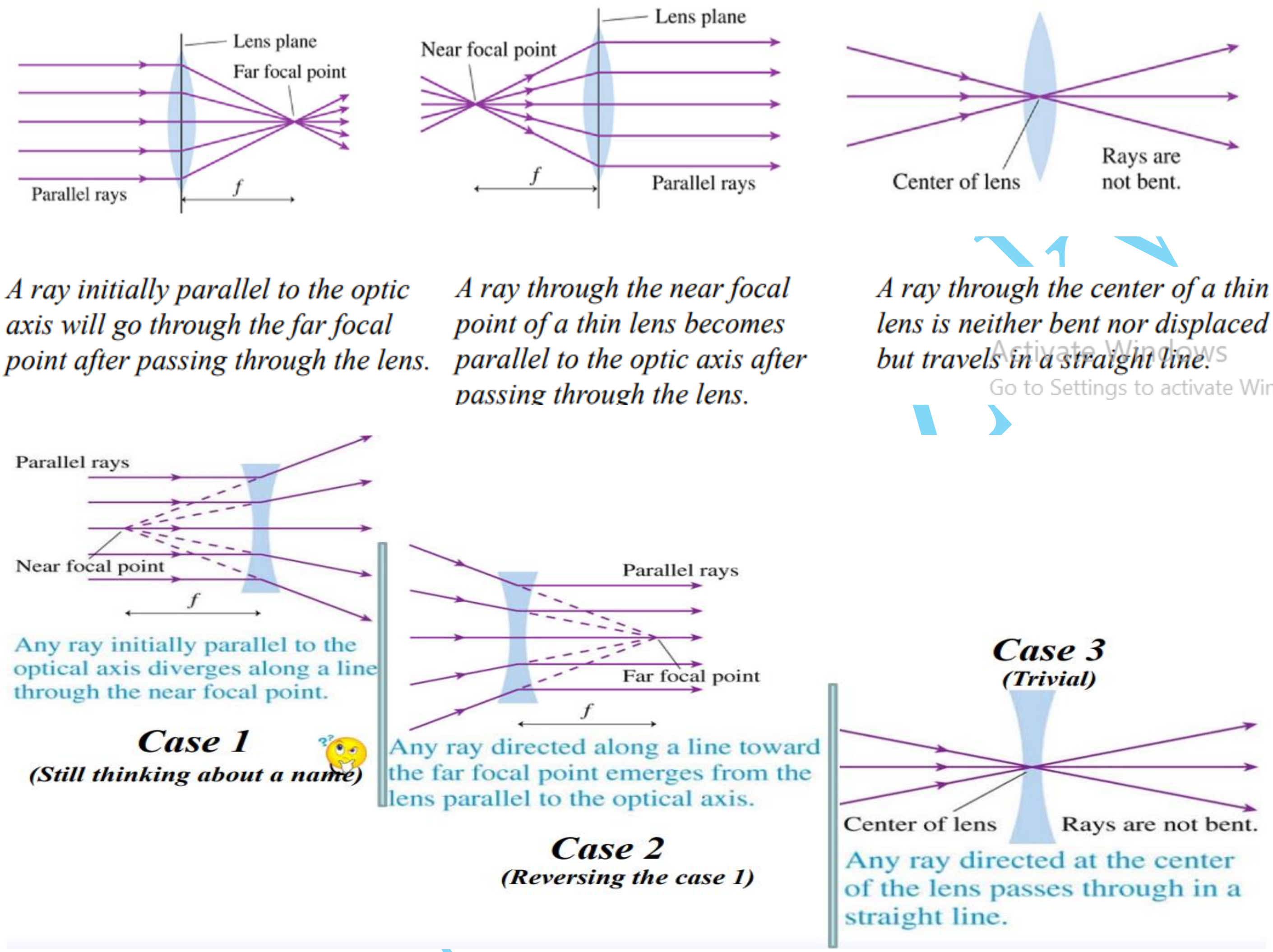
$\frac{\mu 1}{u}+\frac{\mu 2}{v}=\frac{\mu 2-\mu 1}{R}=\frac{\mu 1}{f}$ $\frac{\mu 1}{u}+\frac{\mu 2}{v}=\frac{\mu 2-\mu 1}{R}=\frac{\mu 2}{f'}$ $\frac{\mu 1}{\mu 2}=\frac{f}{f'}$	$m=\frac{-y'}{y}=-\frac{v-R}{u-R}$ $P=\frac{\mu 1}{f}\ ,\quad P'=\frac{\mu 2}{f'}$ $P=\frac{\mu 2-\mu 1}{R}$ $P=\frac{\mu 1}{u}+\frac{\mu 2}{v}$

Thin-Lens:

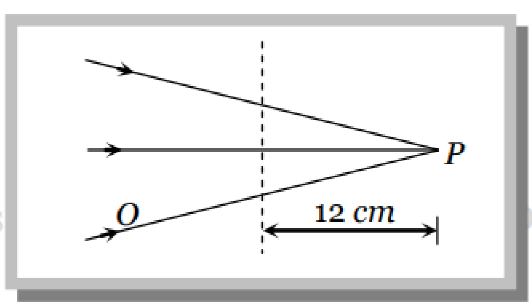




$\frac{\mu 1}{u}+\frac{\mu 1}{v}=(\mu 2-\mu 1)\left(\frac{1}{R1}-\frac{1}{R2}\right)$ $\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{R1}-\frac{1}{R2}\right)$ $\frac{1}{f}=(\mu-1)\left(\frac{1}{R1}-\frac{1}{R2}\right)$	$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $P=\frac{1}{f}$ $f\ in\ (m)\ unit\ to\ get\ P\ in\ diopter($ $M=\frac{y_i}{y_o}\ ,\quad M=-\frac{v}{u}$



Problem1:Figure below shows a beam of light converging at point P. When a concave lens of focal length 16 cm is introduced in the path of the beam at a place O shown by dotted line, the beam converges at a distance v from the lens. Find image distance v.



Sol:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{So} \quad \frac{1}{16}=\frac{1}{-12}+\frac{1}{v} \qquad \therefore v=48cm$$

Thin-Lens Combinations:

Our purpose here is not to become proficient in the intricacies of modern lens design, but rather to gain the familiarity necessary to utilize, and adapt, those lens systems already available commercially:

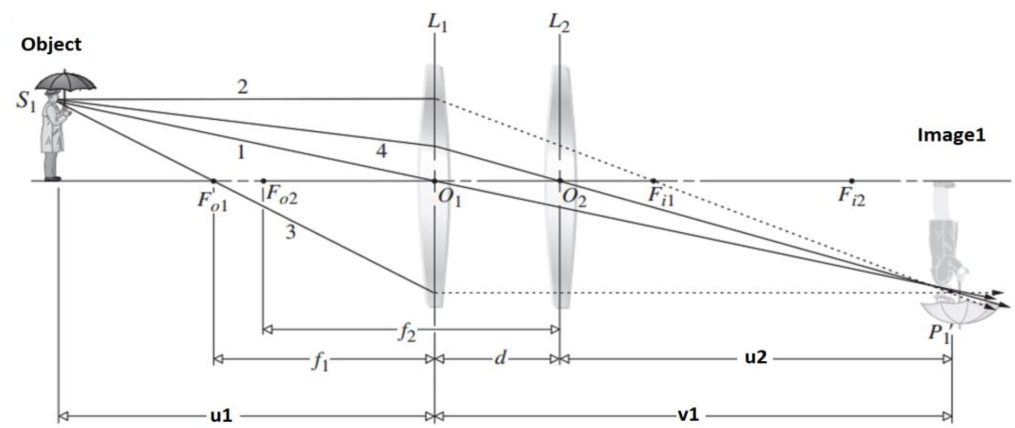


Fig:Two thin lenses separated by a distance smaller than either focal length.

When two lenses are placed co-axially at a distance d from each other as shown in following fig. The relationship between the equivalent focal length (f) with the focal length of the first lens (f1) and the second lens (f2) is as follows:

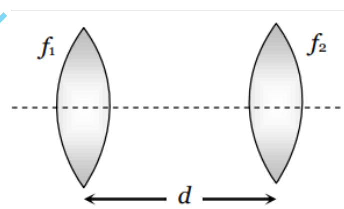


Fig:

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1f2} \qquad ..(1)$$

And

$$P = P1 + P2 - P1P2d$$

Back and Front Focal Lengths:

The distance from the last surface of an optical system to the second focal point of that system as a whole is known as the back focal length, or b.f.l. Similarly, the distance from the vertex of the first surface to the first or object focus is the front local length or f.f.l.:

$$f.f.l. = \frac{f1(d - f2)}{d - (f1 + f2)}$$

$$b.f.l. = \frac{f2(d - f1)}{d - (f1 + f2)}$$

Observe that if $d \rightarrow 0$, that is, the lenses are brought into contact, as in the case of some achromatic doublets

$$f.f.l. = b.f.l. = \frac{f2f1}{f1 + f2} \quad \ldots (2)$$

For two thin lenses in contact the resultant thin lens has an effective focal length, f, such that

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2} \quad and \quad P = P1 + P2 \quad \ldots (3)$$

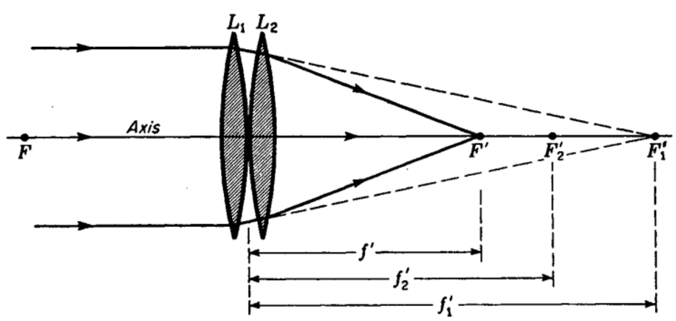


Fig: The power of a combination of thin lenses in contact is equal to the sum of the powers of the individual lenses

This implies that if there are N such lenses in contact:

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2} + \cdots + \frac{1}{fN} \quad \ldots (4)$$

Relative Aperture and *f*-Number: Relative aperture, also known as the f-number, is a fundamental concept in photography and optics. It represents the ratio of the focal length of a lens to the diameter of its entrance pupil. The f-number is expressed as f/N, where "f" is the focal length, and "N" is the aperture (diameter of the entrance pupil). A lower f-number corresponds to a larger aperture, allowing more light to reach the camera sensor and affecting depth of field and exposure in photography.

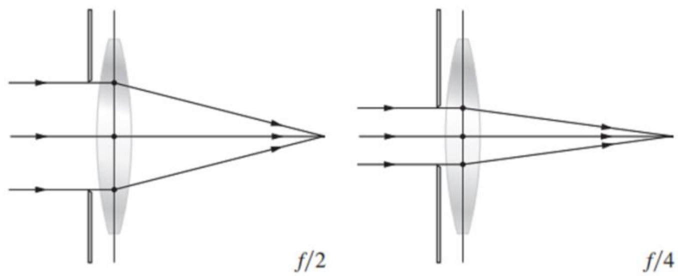


Fig: (a) Stopping down a lens to change the ƒ-number.

The ratio $\frac{D}{f}$ is known as the relative aperture, and its inverse is the focal ratio, or f-number, often written $f/\#$, that is,

$$f/\# = \frac{f}{D} \qquad \dots(5)$$

where $f/\#$ should be understood as a single symbol. For example, a lens with a 25mm aperture and a 50mm focal length has an f-number of 2, which is usually designated $f/2$.

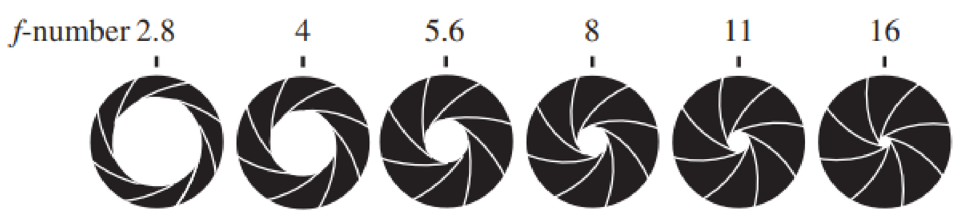


Fig: A camera lens showing possible settings of the variable diaphragm usually located within the lens

Figure above illustrates the point by showing a thin lens behind a variable iris diaphragm operating at either $f/2$ or $f/4$. A smaller f-number clearly permits more light to reach the image plane

Problem2: A convex lens of focal length 40cm is in contact with a concave lens of focal length 25cm. find the power of combination.

Sol. Using

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2}$$

$$\frac{1}{f} = \frac{1}{40} + \frac{1}{-25}$$

$$f = \frac{-200}{3}\ cm = \frac{-2}{3}\ m \quad hence \quad P = \frac{1}{f} = \frac{-3}{2} = -1.5D$$

Problem3: A combination of two thin lenses with focal lengths f1 and f2 respectively forms an image **of distant** object at distance 60cm when lenses are in contact. The position of this image shifts by 30cm towards the combination when two lenses are separated by 10 cm. Find the corresponding values of f1 and f2.

Sol

Initially f=60cm the focal length of combination:

- Hence using

$$\boldsymbol{\frac{1}{f1} + \frac{1}{f2} = \frac{1}{f} = \frac{1}{60}\ and\ \frac{f1f2}{f1+f2} = 60 \quad \ldots(i)}$$

- And by using

$$\frac{1}{f'} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1f2} \quad where\ f' = 30cm\ and\ d = 10cm$$

$$\boldsymbol{\frac{1}{30} = \frac{1}{f1} + \frac{1}{f2} - \frac{10}{f1f2} \quad \ldots(ii)}$$

So from eq(i)and eq(ii) get:

$$\frac{1}{30} = \frac{1}{60} - \frac{10}{f1f2}$$

$$\frac{1}{30} - \frac{1}{60} = -\frac{10}{f1f2}$$

$$\frac{1}{60} = -\frac{10}{f1f2}$$

$$f1f2 = -600$$

From eq(ii) get:

$$\frac{f1f2}{f1+f2} = 60$$

$$\frac{-600}{f1 + f2} = 60$$

$$f1 + f2 = -10 \quad \text{(iii)}$$

Also, difference of focal lengths can written as:

$$f1 - f2 = \sqrt{(f1 + f2)^2 - 4f1f2} = \sqrt{(-10)^2 - 4(-600)} = \sqrt{2500} = 50$$

$$f1 - f2 = 50 \qquad \text{(iv)}$$

From eq(iii) and eq(iv) get:

$$\therefore \boldsymbol{f1 = 20cm\ and\ f2 = -30cm}$$

Problem4: A thin double convex lens has radii of curvature each of magnitude 40cm and is made of glass with refractive index 1.65:find its focal length:

Sol

R1=R and R2=-R;

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R1} - \frac{1}{R2}\right) = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\frac{1}{f} = \frac{2(\mu - 1)}{R}$$

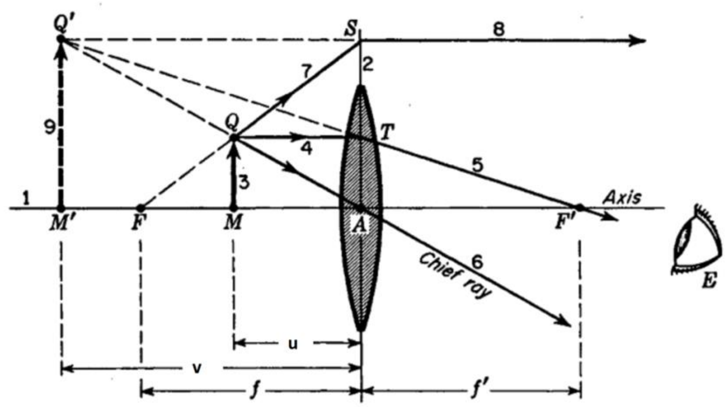
$$f = \frac{R}{2(\mu - 1)} = \frac{40}{2(1.65 - 1)} = 30.7 \text{ cm}$$

Problem5: If an object is located 6cm in front of a lens of focal length 10cm, where will the image be formed?

Sol:

The given quantities are u=6cm, and f =10cm, while the unknown quantities are v and m.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$
$$\frac{1}{6}+\frac{1}{v}=\frac{1}{10}$$
$$\frac{1}{v}=\frac{1}{10}-\frac{1}{6}=\frac{6-10}{60}=\frac{-4}{60}$$
$$v=-15cm$$



The minus sign indicates that the image lies to the left of the lens. Such an image is always virtual. The magnification is obtained by:

$$M=-\frac{v}{u}=-\frac{-15}{6}=+2.5$$

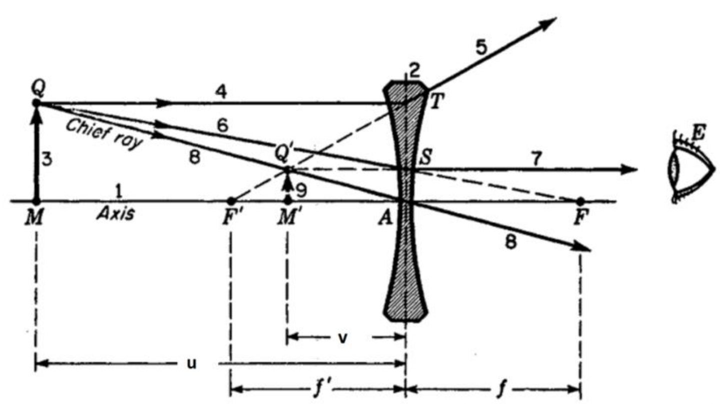
The positive sign means that the image is erect.

Problem6:An object is placed 12cm in front of a diverging lens of focal length 6cm. Find the image.

Sol:

The given quantities are u= + 12cm and f= -6cm, while the unknown quantities are v and m.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$
$$\frac{1}{12}+\frac{1}{v}=\frac{1}{-6}$$
$$\frac{1}{v}=\frac{1}{-6}-\frac{1}{12}=\frac{-2-1}{12}=\frac{-3}{12}$$
$$v=-4cm$$



And

$$M = -\frac{v}{u} = -\frac{-4}{12} = \frac{1}{3} = +0.33$$

The image is therefore to the left of the lens, virtual, erect, and one-third the size of the object.

Hw1:

A 4.00-cm tall light bulb is placed a distance of 45.7cm from a double convex lens having a focal length of 15.2cm. Find the image distance and the image size.

Note: v= 22.8 cm and y_i = -1.99 cm

References

1. Eugene Hecht ,"Optics "fifth edition, © Pearson Education Limited 2017,
2. Francis A. Jenkins , and Harvey E.White, " Fundamental of optics" Fourth Edition, McGraw-Hill Higher Education 1981.
3. N. Subrahmanyam, and Brij Lal ,"A textbook of optics ", S. CHAND & Company LTD. (AN ISO 9001: 2000 company) RAM NAGAR, New Delhui-110 055, 2000.