1. prove $P = n_0 kT$

we have $n_0 = \frac{N_A\, n}{V}$ ....(1)

where n_0 = number density (molecules m^{-3}) , N_A = Avogadro's number (Av = 6.023×10^{23} molecules mol^{-1}), n = number of moles (mole) , v = volume (m^3).

From Equ. 6 $PV = nR^*T$ .... (2)

$$\therefore V = \frac{nR^*T}{P} \quad \text{............(3)}$$

Substituting equation (1) into (3) we obtain:

$$n_0 = \frac{N_A n}{\frac{n\, R^*T}{P}}$$

$$n_0 = \frac{N_A P}{R^*T} \quad \Rightarrow \quad N_A P = n_0\, R^*T \quad \Rightarrow \quad P = \frac{n_0\, R^*T}{N_A}$$

we have $k = \frac{R^*}{N_A}$

So $P = n_0 kT$

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2. prove

$$\frac{dp}{p} = -\frac{Mg}{R^*T}\, dz$$

Hydrostatic Equation $\frac{dp}{dz} = -\rho g$ ....(1)

Using the Ideal Gas Law, we can replace ρ and get the equation for dry air:

$$\frac{dp}{dz} = -g\frac{P}{R_d T} \quad \Rightarrow \quad \frac{dp}{p} = -\frac{g}{R_d T}\, dz \qquad (2)$$

we have $R = \frac{R^*}{M}$ so for dry air $\Rightarrow$ $R_d = \frac{R^*}{M_d}$ .... (3)

Substituting equation (3) into (2) we obtain:

$$\frac{dp}{p} = -\frac{Mg}{R^*T}\, dz$$

Exercise 2: Determine the apparent molecular weight of the Venusian atmosphere, assuming that it consists of 95% of CO2 and 5% N2 by volume. What is the gas constant for 1 kg of such an atmosphere? (Atomic weights of C, O, and N are 12, 16, and 14, respectively.)

Solution:

$$M_d = \frac{\sum_i m_i}{\sum_i \frac{m_i}{M_i}}$$

$$M_d = \frac{1}{\frac{1}{(12+32)\times 0.95 + (28)\times 0.05}} = 43.2\ g$$

$$R_d = 1000\frac{R^*}{M_d} = 1000\frac{8.3145}{43.2} = 192.46\ JK^{-1}Kg^{-1}$$

3. prove $P = \rho R_d T_v$

$$\rho = \rho_d' + \rho_v' \qquad (1)$$

we have $e = \rho_v' R_v T$ so ⟶ $\rho_v' = \frac{e}{R_v T}$ ....(2)

and we have $P_d' = \rho_d' R_d T$ so ⟶ $\rho_d' = \frac{P_d'}{R_d T}$ ....(3)

Substituting equation (2 and 3) into (1) we obtain

$$\rho = \frac{P_d'}{R_d T} + \frac{e}{R_v T} \quad \dots\dots\dots..(4)$$

$$\because \ P = P_d' + e \quad so \implies \quad P_d' = P - e \ \dots\dots\dots(5)$$

Substituting equation (5) into (4) we obtain

$$\rho = \frac{P - e}{R_d T} + \frac{e}{R_v T} \qquad \dots\dots\dots..(6)$$

we have $\frac{R_d}{R_v} = \varepsilon$ so $R_v = \frac{R_d}{\varepsilon}$

$$\rho = \frac{P - e}{R_d T} + \frac{e}{\frac{R_d}{\varepsilon}\ T} \qquad \implies \rho = \frac{P - e}{R_d T} + \frac{\varepsilon\, e}{R_d\ T}$$

$$\implies \rho = \frac{P}{R_d T} - \frac{e}{R_d T} + \frac{\varepsilon\, e}{R_d\ T}$$

$$\rho = \frac{P}{R_d T}\left[1 - \left(\frac{e}{P} + \frac{\varepsilon e}{P}\right)\right] \quad \implies \rho = \frac{P}{R_d T}\left[1 - \frac{e}{P}(1 - \varepsilon)\right]$$

$$P\left[1 - \frac{e}{P}(1 - \varepsilon)\right] = \rho R_d T$$

$$P = \rho R_d \frac{T}{\left[1 - \frac{e}{P}(1 - \varepsilon)\right]}$$

$$T_v = \frac{T}{1 - \frac{e}{P}(1 - \varepsilon)}$$

$$P = \rho R_d T_v$$