**8. Homomorphism, Examples and Basic Concepts**

**Definition(8-1):**Let be two groups and be a mapping, then is called a homomorphism iff .

**Example(8-2):** Let . Is a homo. ?

**Solution:** let

thus, is a homo.

**Example(8-3):** Let . Is a homo. ?

**Solution:** let

We have

Therefore, is not a homo.

**Example(8-4):** Let . Is a homo. ? (**Homework**)

**Example(8-5):** Let . Is a homo. ? (**Homework**)

**Example(8-6):** Let . Is a homo. ?

**Solution:**

We have

Therefore, is not a homo.

**Example(8-7):** Let

. Is a homo. ?

**Solution:** let

1. If

Therefore, is a homo.

**Example(8-8):** Let . Is a homo. ?

**Solution:** let

We have therefore, is a homo.

**Example(8-9):** Let . Is a homo. ?

**Solution:** let

Therefore, is a trivial homo.

**Example(8-10):** Let and . Is a homo. ?

**Solution:** let

We have Therefore, is a natural homo.

**Definition(8-11):** Let be a mapping, then

1. is called a monomorphism (mono.) iff is a homo. and one to one.
2. is called an epimorphism (epi.) iff is a homo. and onto.
3. is called an isomorphism (iso.) iff is a homo., one to one and onto.

**Definition(8-12):** Any two groups are isomorphic iff there is an isomorphism map between them and denoted by .

This means, and is an isomorphism.

**Example(8-13):** Let , show that .

**Solution:** define

Homo.? let

is a homo.

One to one? let , to prove

is a one to one

Onto? is an onto

is an isomorphism

**Theorem(8-14):** Let be an isomorphism, then

1. such that the identity of .

**Proof:** let

Let

.

**Proof:** let

let

.

1. If is a subgroup of a group , then is a subgroup of .

**Proof:**

Let , to prove

1. If is a subgroup of , then is a subgroup of .

**Proof:**

Let , to prove

is a subgroup of .

1. If and an onto, then .

**Proof:** let , to prove

and is an onto

and

.

1. If , then .

**Proof:**  is a subgroup of , to prove

Let

and

.

**Theorem(8-15):** The relation of isomorphic is an equivalent.

**Proof:** Reflexive: to prove and is a homomorphism, one to one and onto, thus is an isomorphism.

Symmetric: let , to prove , is an isomorphism, is a bijective

is an one to one and onto, to prove is a homomorphism, let is an onto

Thus, is a homomorphism, is an isomorphism,

.

Transitive: let and , to prove

is an isomorphism, is an isomorphism. is a bijective. Let

Hence, is a homomorphism is an isomorphism

is an equivalent relation.

**Theorem(8-16):** Prove that

1. Every two finite cyclic groups of the same order are isomorphic.

**Proof:** let are two finite cyclic groups,

is a cyclic

is a cyclic

Define let mod, thus is a map.

Let mod is a one to one.

is an onto.

is a homomorphism is an isomorphism.

1. Every finite cyclic group is an isomorphism to.

**Proof:** let be a finite cyclic group

1. if is not an onto
2. if

define let mod is a map.

Let mod is an one to one.

is a homomorphism.

is an onto is an isomorphism.

1. Every two infinite cyclic group are isomorphic.

**Proof:** let are infinite cyclic groups.

Define

* is a map (**Homework**)
* is an one to one (**Homework**)
* is an onto (**Homework**)
* is a homomorphism (**Homework**)

1. Every infinite cyclic group is an isomorphic to .

**Proof:** since is a cyclic

Define (**check**)

**Definition(8-17):** Let be a group, define

1. Homis a homomorphism
2. Autis an isomorphism

**Theorem(8-18):** Let be a group, then

1. Aut is a group.

**Proof:** 1,2 and 3 (**check**)

Inverse: let , is an isomorphism, since is a bijective and since is an isomorphism is an isomorphism and Aut is a group.

1. Aut is a subgroup of Symm.

**Proof:** Autis an isomorphism

Symmis a bijective

Aut, since is an isomorphism

Aut Symm and Aut is a group

Aut is a subgroup of Symm.

**Definition(8-19):** Let be a group and . Define , then is called an inner automorphism of and Inn or I.

**Theorem(8-20):** Let be a group and , then:

1. is an isomorphism map.

**Proof:**

Thus, is a homomorphism.

Let is an one to one.

is an isomorphism map.

1. is a subgroup of Aut.

**Proof:** Iis an isomorphism

Autis an isomorphism

Closure: let

Inverse: let , is a subgroup of Aut.

**Proof:** Iis an isomorphism

Autis an isomorphism

Let .

**Definition(8-21):** Let be a group homomorphism, then the kernel of denoted by ker and defined by ker

**Example(8-22):** let , find ker.

**Solution:**  is a homomorphism (**check**) ker an exist,

ker

**Example(8-23):** Let is a trivial homomorphism, find ker.

**Solution:**is a homomorphism keris an exist.

ker.

**Example(8-24):**let , find ker.

**Solution:** is a homomorphism (**check**)

Ker(mod 3).

**Theorem(8-25):** Let be a group homomorphism, then:

1. Ker is a subgroup of .

**Proof:** kerker.

Let ker, ker Ker is a subgroup of .

1. Ker

**Proof:** Ker is a subgroup of .

Let KerKerKer.

1. Ker iff is an one to one.

**Proof:**  suppose that Ker

Let

Ker

let Ker

Ker.