Week Two
Introduction to number base conversion

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Addition

- Decimal Addition

$$
\begin{array}{cccc}
 & 1 & 1 & \\
 & & 5 & 5 \\
+ & & 5 & 5 \\
\hline
 & 1 & 1 & 0
\end{array}
\qquad \textbf{Carry}
$$

= ***Ten*** $\geq$ ***Base***

➔ **Subtract a Base**

Binary Addition

- Column Addition

$$
\begin{array}{cccccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & & & \\
 & & 1 & 1 & 1 & 1 & 0 & 1 & & = 61 \\
+ & & & 1 & 0 & 1 & 1 & 1 & & = 23 \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & & = 84
\end{array}
$$

$$\geq (2)_{10}$$

Binary Subtraction

- Borrow a “Base” when needed

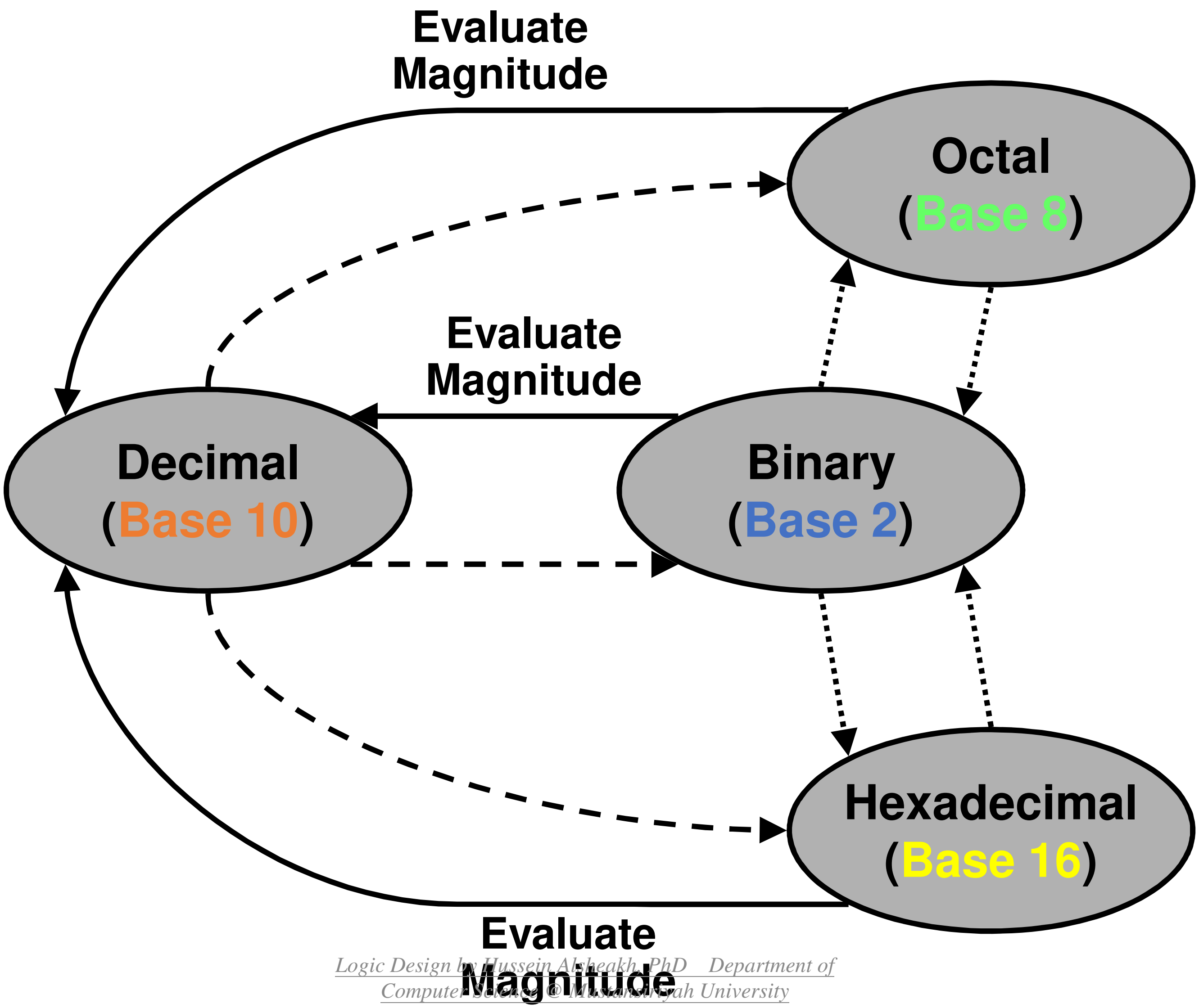
$$
\begin{array}{cccccccccl}
 & & 1 & & & 2 & & & & = (10)_2 \\
 & 0 & \cancel{2} & 2 & 0 & 0 & 2 & & & \\
 & \cancel{1} & 0 & 0 & \cancel{1} & \cancel{1} & 0 & 1 & & = 77 \\
- & & & 1 & 0 & 1 & 1 & 1 & & = 23 \\
\hline
 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & & = 54
\end{array}
$$

Binary Multiplication

- Bit by bit

				1	0	1	1	1
x					1	0	1	0
				0	0	0	0	0
			1	0	1	1	1	
		0	0	0	0	0		
	1	0	1	1	1			
	1	1	1	0	0	1	1	0

Number Base Conversions



Decimal (*Integer*) to Binary Conversion

- Divide the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: $(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

MSB **LSB**

Decimal (*Fraction*) to Binary Conversion

- Multiply the number by the ‘Base’ (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

		Integer		Fraction	Coefficient
0.625	* 2 =	1	.	25	$a_{-1} = 1$
0.25	* 2 =	0	.	5	$a_{-2} = 0$
0.5	* 2 =	1	.	0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
$175 / 8 =$	21	7	$a_0 = 7$
$21 / 8 =$	2	5	$a_1 = 5$
$2 / 8 =$	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer		Fraction	Coefficient
$0.3125 * 8 =$	2	.	5	$a_{-1} = 2$
$0.5 * 8 =$	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_8 = (0.24)_8$

Binary – Octal Conversion

- $8 = 2^3$
- Each group of 3 bits represents an octal digit

Example:

Assume Zeros

$$(\ 1\ 0\ 1\ 1\ 0\ .\ 0\ 1\ )_2$$

$$(\ 2\ \ 6\ .\ 2\ )_8$$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works both ways (*Binary* to *Octal* & *Octal* to *Binary*)

Binary – Hexadecimal Conversion

- $16 = 2^4$
- Each group of 4 bits represents a hexadecimal digit

Example:

Assume Zeros

$(10110.01)_2$

$(1\ 6\ .\ 4)_{16}$

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Works both ways (*Binary* to *Hex* & *Hex* to *Binary*)

Octal – Hexadecimal Conversion

- Convert to Binary as an intermediate step

Example:

$$(2\quad 6\quad .\quad 2)_8$$

Assume Zeros

$$(0\,1\,0\,1\,1\,0\,.\,0\,1\,0)_2$$

Assume Zeros

$$(1\quad 6\quad .\quad 4)_{16}$$

Works both ways (*Octal* to *Hex* & *Hex* to *Octal*)

HW
Convert the following, and Show your steps

- (527) 10 to BCD
- 10111011 to octal
- 1011011101 to hexadecimal
- Convert the following Octal number, 330.93758 to Decimal
- Convert the following binary number, 101.012 to Decimal
- Convert the following binary number, 1E5.7A16 to Decimal