Lecture 3
Complements

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Quiz1

Convert (7 1 6 . 2 5)8 → (X)16 ?

Sol:

(7 1 6 . 2 5) 8 = (1 1 1 0 0 1 1 1 0 . 0 1 0 1 0 1) 2

(0 0 0 1 1 1 0 0 1 1 1 0 . 0 1 0 1 0 1 0 0)2 = (1 C E . 5 4)16

Converting to Other Systems

- $(X)5 \rightarrow (X)8$ or $(X)2 \rightarrow (X)7$

في هذه الحالة يتم التحويل الى النظام العشري اولا ثم يحول الرقم الناتج الى النظام المطلوب.

<u>Ex</u> Convert $(1\ 1\ 0\ 1\ 1\ 0)_2 \rightarrow (X)_7$?

Sol:

1 - $(1\ 1\ 0\ 1\ 1\ 0) = 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 1 \times 2^5$

$= 0 + 2 + 4 + 0 + 16 + 32 = (54)_{10}$

2 –

	Rem.
$54 \div 7 = 7$	5 ↑
$7 \div 7 = 1$	0
$1 \div 7 = 0$	1

$(1\ 1\ 0\ 1\ 1\ 0)_2 = (54)_{10} = (1\ 0\ 5)_7$

Binary Division

Binary arithmetic operations

Addition	Subtraction	Multiplication	Division
0 + 0 = 0	0 – 0 = 0	0 X 0 = 0	0 ÷ 0 = 0
0 + 1 = 1	1 – 0 = 1	0 X 1 = 0	0 ÷ 1 = 0
1 + 0 = 1	0 – 1 = 1 borrow 1	1 X 0 = 0	1 ÷ 1 = 1
1+ 1 = 0 carry 1	1 – 1 = 0	1 X 1 = 1	1 ÷ 0 = Overflow

- Perform the following operations:
- 1 - (1 1 1 1)2 ÷ (1 0 1)2
- 2 - (1 1 0 0 1)2 ÷ (1 0 1)2
- 3 - (1 0 1 1 0)2 ÷ (1 0)2
- 4 - (1 1 0 1 1)2 ÷ (1 0 0)2
- 5 - (1 1 1 0 1)2 ÷ (1 1 0 0)2
- 6 - (1 0 0 1 0 0 0 1)2 ÷ (1 0 11)2
- 7 - (1 0 1 0 . 0 1)2 ÷ (1 . 1)2

Sol:

1-

```
        1 1
101 | 1 1 1 1
      1 0 1
      0 1 0 1
        1 0 1
        0 0 0
```

2-

```
          1 0 1
1 0 1 | 1 1 0 0 1
        1 0 1
        0 0 1 0 1
            1 0 1
            0 0 0
```

Exercises

- Ex Perform the following operations:-
  1 - (4 7 1) 8 + (6 3 5) 8 = (1 3 2 6) 8
  2 - (2 A 4) 16 *+ (C B 4)* 16 *= (F 5 8)* 16
  3 - (4 0 5) 8 - (2 6 7) 8 = (1 1 6) 8
  4 - (A 8 5) 16 - (5 D 4) 16
  5 - (6 5 2) 12 - (4 8 0) 12
  6 - (1 4 5 A 2) 16 x (1 . 3) 16
  7 - (3 4 2) 8 + (12)10
  8 - (3 2 2 . 2) 5 - (4 3 . 4) 5
  10- (5 3 7 . 4) 10 + (1 1 0 0 0 . 1 1) 2
  11- (5 A 4) 11 X (2 . 3) 11
  12- (1 0 0 1 1 . 0 1) 2 - (1011 . 11) 2
  14- (1 A B . 8) 16 - (2 5 3 . 9) 10
  15- (1 1 1 . 0 1) 2 X (1 . 0 1) 2

- Convert the following numbers from a given base to the base indicated?
  1- (1 1 0 1 0 1)2 → (X)3
  2- (A77 . C5)16 → (X) 2
  3- (1 0 1 0 1 1) 2 → (X) 7
  4- (6 A . 2 0 5) 16 → (X) 8
  5- (1 2 3) 4 → (X) 5
  6- (6 7 0 1 . 2 5 4) 8 → (X) 16
  7- (325.14)8 → (X) 10
  8- (1A . 4) 16 → (X) 10
  9- (67.33)10 → (X)2

Counting in number systems:-

The counting in any system is done by starting with the first digit in the system (0) until the maximum digit of the system is reached, and then the counting is continued using 2 digits, and so on.

Ex Write the first 17 digits in base 8?

Sol:

(0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20)

Ex Write the first 30 digits in a Hexadecimal system?

Sol:

(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,1A, 1B,1C, 1D)

Ex Write 10 digits in base 8 starting with decimal 5.

Sol:

(5, 6, 7, 10. 11, 12, 13, 14, 15, 16)

Complements

- Complements are used in digital computer for simplifying the subtraction operation and for logical manipulations. There are two types of complements for each base-*r* system:
  - Radix complement (r's complement). *(r^n –N)*
  - Diminished radix complement *(r^n –1) – N*'s complement.
- **Radix Complement - r’s Complement**
- Ex: Find the r's comp. of the following numbers:

1 – (5 2 5 2 0)10 2 - (3 2 6 7)10 3 - (2 5. 6 3 9)10

4 - (1 0 1 1 0 0)2 5 - (8 7 6 5)11 6 - (A 0 9 0)16

- Sol:
- 1- The 10's comp. of (5 2 5 2 0)10 is (47 4 8 0)
- 2- The 10's comp. of (3 2 6 7)10 is (6 7 3 3)
- 3- The 10's comp. of (2 5. 6 3 9)10 is (7 4. 3 6 1)
- 4- The 2's comp. of (1 0 1 1 0 0)2 is (0 1 0 1 0 0)
- 5- The 11's comp. of (8 7 6 5)11 is (2 3 4 6)
- 6 - The 16's comp. of (A 0 9 0)16 is (5 F 7 0)

Complements

- Complements are used in digital computer for simplifying the subtraction operation and for logical manipulations. There are two types of complements for each base-*r* system:
  - Radix complement (r's complement). $(r^n - N)$
  - Diminished radix complement (r – 1)'s complement.
- **Diminished Radix Complement - (r-1)'s Complement**
  - Given a number *N* in base *r* having *n* digits, the $(r-1)$'s complement *of N* is defined as:

$$(r^n - 1) - N$$

- **Example for 6-digit <u>decimal</u> numbers**:
  - 9's complement is $(r^n - 1) - N = (10^6 - 1) - N = 999999 - N$
  - 9's complement of 546700 is 999999–546700 = 453299
- **Example for 7-digit <u>binary</u> numbers:**
  - 1's complement is $(r^n - 1) - N = (2^7 - 1) - N = 1111111 - N$
  - 1's complement of 1011000 is 1111111–1011000 = 0100111
- **Observation:**
  - Subtraction from $(r^n - 1)$ will never require a borrow
  - Diminished radix complement can be computed digit-by-digit
  - For binary: 1 – 0 = 1 and 1 – 1 = 0

The (r -1)' s complement.

- Ex: Find the (r -1)'s comp. of the following numbers:

1 – (5 2 5 2 0)10 2 - (3 2 6 7)10 3 - (2 5. 6 3 9)10

4 - (1 0 1 1 0 0)2 5 - (8 7 6 5)11 6 - (A 0 9 0)16

- Sol:
- 1- The 9's comp. of (5 2 5 2 0)10 is (47 4 7 9)
- 2 The 9's comp. of (3 2 6 7)10 is (6 7 3 2)
- 3 - The 9's comp. of (2 5. 6 3 9)10 is (7 4. 3 6 0)
- 4 - The 1's comp. of (1 0 1 1 0 0)2 is (0 1 0 0 1 1)
- 5 - The 10's comp. of (8 7 6 5)11 is (2 3 4 5)
- 6 - The 15's comp. of (A 0 9 0)16 is (5 F 6 F)

Ex Find the 1's and 2's comp. of (1 0 1 1 0. 1 0 0)2 ?

Complements

- Complements are used in digital computers to simplify the **subtraction operation** and for logical manipulation.
- 1's Complement (*Diminished Radix* Complement)
  - All '0's become '1's
  - All '1's become '0's

  Example $(10110000)_2$

  ⇨ $(01001111)_2$

  If you add a number and its 1's complement …

```
  1 0 1 1 0 0 0 0
+ 0 1 0 0 1 1 1 1
-----------------
  1 1 1 1 1 1 1 1
```

1's Complement (Diminished Radix Complement)

- **Binary numbers Complement:**
- 1's complement= $(r^n - 1) - N$
- where

  n : number of bits

  N: binary number

  r : system base
- Simply the 1's complement of binary number is the number we get by changing each bit (0 to 1) and (1 to 0).
- Example: the first complement of $(101100)_2$
- Solution:

binary number	101100
1's complement	010011

Decimal numbers Complement:

$(r^n - 1) - N$

In base 10: Finding 5 digits the 9's complement of 1357.

We have n = 5; r = 10, N = 1357.

Result = $(10^5 - 1) - 1357$ = 98642

$2^{'s}$ Complement (Radix Complement)

- **Binary numbers Complement**
- The equation is:
- 2's complement = $r^n - N$

- Simply the 2's complement is equal to 1's complement added by one.
- Example: find the 2's complement of $(101101)_2$
- Solution:

binary number	101101
1's complement	010010
2's complement	010010
	+ 1

	010011

Decimal numbers Complement

- Radix complement
  - Defined as $r^n - N$;
- **In base 10: Finding 5 digits the 10's complement of 1357.**
- **Result = $10^5 - 1357$ = 98643**

Unsigned vs Signed Numbers

- Unsigned
  - All bits are used to show the magnitude of the number.
  - All numbers are considered to be positive
- Signed
  - Positive and Negative

Signed Numbers

- There are three basic ways to designate the sign of a number.
  - Sign and magnitude
  - Radix complement (1's complement)
  - Radix-1’s complement (2's complement)

Why use complement?

-simplifying the subtraction operation by adding a complement of that number instead of subtraction for that number

$15_{10} – 4_{10} = 15_{10}$-(complement of 4_{10}) = 11_{10}

A-B= A+(-B) more simple fore Hardware design

Sign and Magnitude

- What is taught in school.
- A value with a sign in front of it
- How does it work in Binary?
- Pretty much the same way as Decimal
- By convention a sign bit is used.
  - 0 -→ positive
  - 1 -→negative

a = 1 111 ?

→a = 15_{10} (if unsigned)

Or -→ a = -7 (if signed).

<u>Ex</u>: Represent $(+12)_{10}$, $(-12)_{10}$ in signmagnitude, 1's and 2's complement?

<u>Sol :</u>

	Signmagnitude	1's comp.	2's comp.
+12	0 1100	0 1100	0 1100
-12	1 1100	1 0011	1 0100

Subtraction with Complements

- The subtraction of two *n*-digit unsigned numbers *M* – *N* in base *r* can be done as follows:

1. Find the r s comp. of N.
2. Add M to the result of step 1.
3. Inspect the result obtained in step 2 for an end carry:-
   a. If an end carry occurs, discard it.
   b. If an ***end carry does not occur***, take the r's comp. of the number obtained in step 2 and place negative sign in front it.

Ex Subtract (7 2 5 3 2 - 3 2 5 0)10

using 10's comp.?
Sol: M = 7 2 5 3 2 , N = 0 3 2 5 0
1 – The 10 s comp. of N is (9 6 7 5 0)

```
2 -   7 2 5 3 2
    + 9 6 7 5 0
      ----------------

     1 6 9 2 8 2
```

3 – The result is (6 9 2 8 2)10

Ex: Subtract $(3250 - 72532)_{10}$ using r's comp?

Sol : M=03250 , N=72532

1- The l0 s comp. of N is (2 7 4 6 8)

```
2-      0 3 2 5 0
      + 2 7 4 6 8
    -------------------
        3 0 7 1 8       there is no carry
```

3- The l0's comp .of (3 0 7 1 8) is (6 9 2 8 2)

The result is $(- 6\ 9\ 2\ 8\ 2)_{10}$

Complements

Ex: Subtract $(1\,0\,0\,0\,1\,0\,0)_2$ - $(1\,01\,0\,1\,0\,0)_2$ using 2's comp.?

Sol: M=1 0 0 0 1 0 0 , N = 1 0 1 0 1 0 0

1- The 2 s comp of N is (0 1 0 1 1 0 0)

2-
$$\begin{array}{r} 1\,0\,0\,0\,1\,0\,0 \\ +\,0\,1\,0\,1\,1\,0\,0 \\ \hline 1\,1\,1\,0\,0\,0\,0 \end{array}$$
there is no carry

3 - The 2's comp. of $(1\,1\,1\,0\,0\,0\,0)_2$ is $(0\,0\,1\,0\,0\,0\,0)$

The result $(-1\,0\,0\,0\,0)_2$

- Example
  - 32 – 3250.

$$\begin{aligned} M &= \quad 72532 \\ \text{10's complement of} \quad N &= \underline{+96750} \\ \text{Sum} &= \quad 169282 \\ \text{Discard end carry } 10^5 &= \underline{-100000} \\ \text{Answer} &= \quad 69282 \end{aligned}$$

r^n

- Example
  - Using 10's complement, subtract 3250 – 72532.

$$\begin{aligned} M &= \quad 03250 \\ \text{10's complement of} \quad N &= \underline{+27468} \\ \text{Sum} &= \quad 30718 \end{aligned}$$

There is no end carry.

Therefore, the answer is – (10's complement of 30718) = – 69282.

Subtraction using (*r* – 1)'s complement

Example Given the two binary numbers *X* = 1010100 and *Y* = 1000011, perform the subtraction (a) *X* – *Y* ; and (b) *Y* – *X*, by using 1's complement.

(a) $X - Y = 1010100 - 1000011$

$X =$	1010100
1's complement of $Y =$	± 0111100
Sum =	10010000
End-around carry =	+ 1
Answer. $X - Y =$	0010001

(b) $Y - X = 1000011 - 1010100$

$Y =$	1000011
1's complement of $X =$	+ 0101011
Sum =	1101110

There is no end carry, Therefore, the answer is Y – X = – (1's complement of 1101110) = – 0010001.

Subtraction using $(r - 1)$'s complement

- The procedure for subtracting two numbers (M – N) with (r -1) s comp. is done as follow:

1. Find the (r-1)'s comp. of N.
2. Add M to the result of step 1
3. Inspect the result obtained in step 2 for an end carry:-
   a. If an end carry occurs, add 1 to the least significant digit.
   b. If an ***end carry does not occur***, take the (r-1)'s comp. of the number obtained in step 2 and place negative sign in front it.

Ex Subtract $(3250\text{-}72532)_{10}$ using 9's comp?

Ex Find 83-27 using 1's comp?

Sol: M = 83 = (1 0 1 0 0 1 1) 2 , N = 27 = (0 0 1 1 0 1 1) 2

1- The 1 s comp .of N is (1 1 0 0 1 0 0)

2-
```
   1 0 1 0 0 1 1
 + 1 1 0 0 1 0 0
 ---------------
 1 0 1 1 0 1 1 1    there end carry
```

3-
```
   0 1 1 0 1 1 1
 +             1
 ---------------
   0 1 1 1 0 0 0    the result is 1 1 1 0 0 0
```

Ex: Perform the operation (A B C E F) 16 - $(4\ 8\ F\ 9\ D)_{16}$ using (r – 1)'s comp.?

Sol: M = A B C E F , N = 4 8 F 9 D

1- The 15 s comp. of N is (B 7 0 6 2)

2-
```
     A B C E F
   + B 7 0 6 2
   -----------
   1 6 2 D 5 1    there is end carry
```

3-
```
     6 2 D 5 1
   +         1
   -----------
     6 2 D 5 2    the result is (6 2 D 5 2)16
```

Complements

- Example
  - Given the two binary numbers *X* = 1010100 and *Y* = 1000011, perform the subtraction (a) *X* – *Y* ; and (b) *Y* – *X*, by using 2's complement.

(a)

X =	1010100
2's complement of Y =	+0111101
Sum =	10010001
Discard end carry 2^7 =	–10000000
Answer. X – Y =	0010001

(b)

Y =	1000011
2's complement of X =	+ 0101100
Sum =	1101111

There is no end carry. Therefore, the answer is Y – X = – (2's complement of 1101111) = – 0010001.

Practice

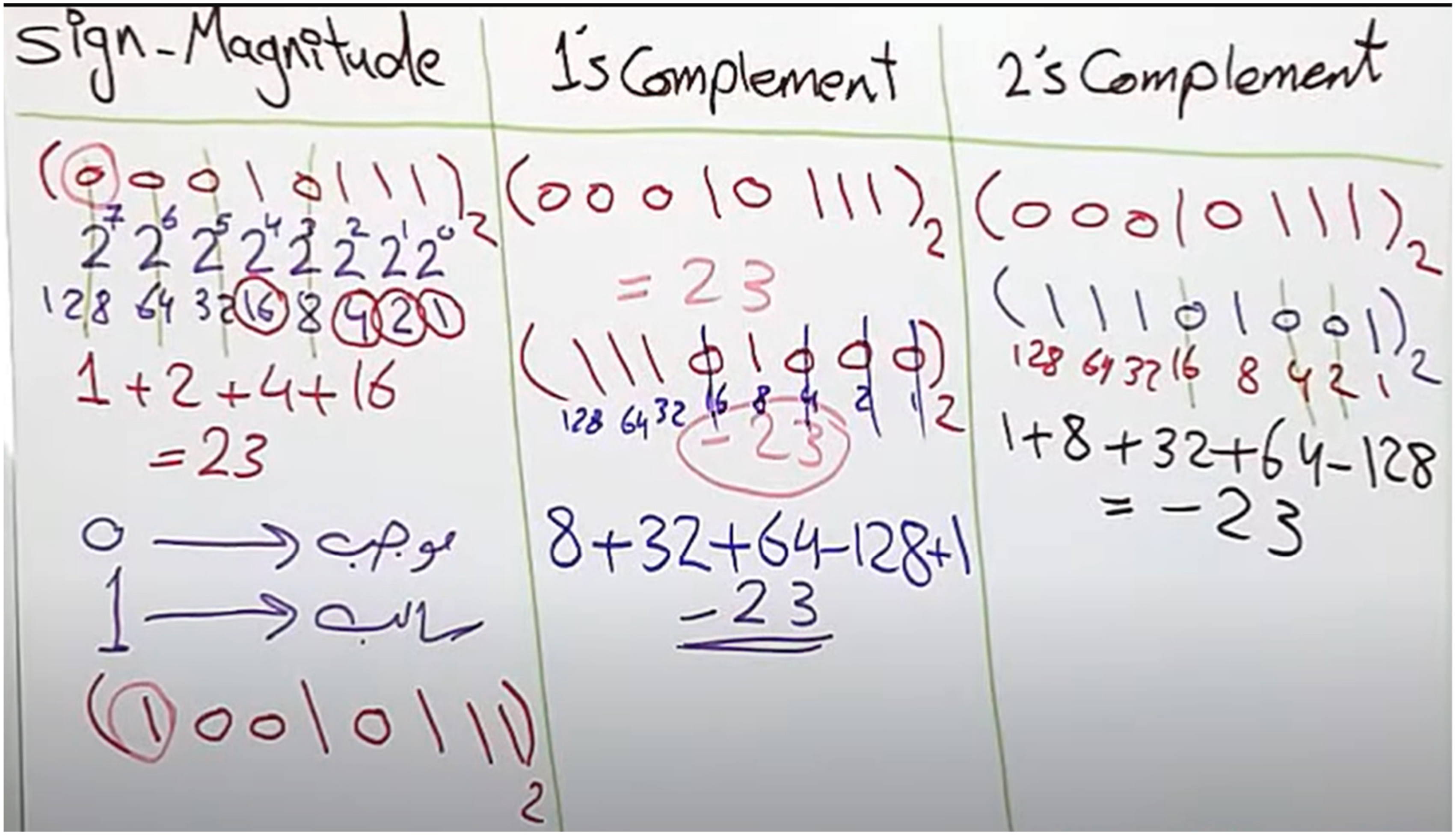
- **Finding 8 digits the 1's complement of 01111000.**
  - **Answer: ?**
- **Finding 8 digits the 2's complement of 0111 1000.**
  - **Answer: ?**

Practice

- **Finding 8 digits the 1's complement of 01111000.**
  - **Answer: 1000 0111**
- **Finding 8 digits the 2's complement of 0111 1000.**
  - **Answer: 1000 0111+**

1 =

1000 1000



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Complements Summary

- The complement of the complement returns the original number
- If there is a radix point
  – Calculate the complement as if the radix point was not there.
- **Used in computers to perform subtraction**

- 1's complement
  – Is the interim step towards 2's complement
  – Problem: Two values of 0.
- + 0: 0000 0000, let's flip all bits
  - 0: 1111 1111
- 2's complement
  – Most CPU's today use 2's complement.
  – Only one value 0: 0000 000

Assignment 1

Write a report to show the difference between Digital and Analog in term of concept, Applications, Why, When, What?