Cloud Physics Lab

LAB 6 : Growth of Ice Crystals III

Growth by Accretion

Introduction:

Accretion describes the collection of supercooled liquid droplets by a falling ice crystal. The supercooled liquid droplets freeze on contact with the crystal. This is how graupel and hail are formed. In this lab, student will explore the growth of ice crystal by accretion

Objective:

a) Plot and study the growth of ice crystal by accretion for different liquid water contents.
b) Plot and study the distance that ice crystal needs to fall inside the cloud for given liquid water contents to grow to a certain size by accretion.

Theory:

The equation for growth by accretion is also very similar to that for collision-coalescence and aggregation, and is written as:

$$\frac{dm}{dt} = \overline{E} M_{liq} \pi R^2 u(R) \quad (1)$$

where M_{liq} is the cloud liquid water content. In this equation, the speed of the smaller droplets is ignored. Over bar E is the average collection efficiency, R is the effective radius of large crystal, and *u(R)* is speed of large crystal.

For graupel particle the relation between the mass and radius is approximately:

$$m = 0.562 R^3 \quad \text{(CGS units)} \quad (2)$$

and the dependence of graupel fall speed on size may be approximated as:

$$u = 520 R^{0.6} \quad \text{(CGS units)} \quad (3)$$

From equations (1) through (3) it can be shown that the growth rate of spherical graupel is:

$$\frac{dR}{dt} = 123.37 \overline{E} M_{liq} \pi R^{0.6} \quad (4)$$

Note that the units is CGS. Equation (4) can be integrated from R_o to R_f to yield:

$$R_f = \left(R_o^{0.4} + 123.37 \overline{E} M_{liq} \pi \Delta t \right)^{2.5} \quad (5)$$

To find distance traveled by the graupel inside the cloud:

$$\frac{dz}{dt} = w - u(R) \quad (6)$$

From equation (1),

$$dm = \overline{E}M_{liq}\pi R^2 u(R)dt \tag{7}$$

and from equation (2)

$$dm = 0.562 \times 2R^2 dR = 1.686R^2 dR \tag{8}$$

From equations (7) and (8) we obtain:

$$u(R) = \frac{1.686}{\overline{E}M_{liq}\pi}\frac{dR}{dt} \tag{9}$$

Substituting u(R) from equation (9) results in:

$$\frac{dz}{dt} = w - \frac{1.686}{\overline{E}M_{liq}\pi}\frac{dR}{dt} \tag{10}$$

Integrating equation (10) gives:

$$z = wt_f - \frac{1.686}{\overline{E}M_{liq}\pi}\left(R_f - R_o\right) \tag{11}$$

Materials and Procedures:

1. Run the Matlab script **Lab12a.m** to plot the growth rate of ice crystal by accretion.
2. Run the Matlab script **Lab12b.m** to plot the fall distance of ice crystal inside the cloud needed to grow to a given radius.

Analysis and Conclusions:

1. Use figure 1 to describe the behavior of ice crystal growth by accretion for different water contents.
2. Use figure 2 to describe how long ice crystal has to fall so that it can grow by accretion to a given radius.

Questions:

1. By completing this Lab, what did you learn about the growth of ice crystal by accretion?
2. In fig. 1, explain why the liquid water content has small effect on the growth rate of small crystals?
3. Fig. 2 shows that crystal reaches different peaks at different liquid water contents. Can you explain the reason for this behavior?