Boolean Algebra and Logic Gates

1 Definition
Useful mathematical system for specifying and transforming logic functions. We study Boolean algebra as the foundation for designing and analyzing digital systems.

Boolean Function Evaluation
A binary variable can take the value of 0 or 1. **A Boolean function** is an expression formed with binary variable, the two binary operators OR and AND, the NOT operator, parentheses, and equal sign. The value of a function maybe **0** or **1**, depending on the values of variables present in the Boolean function or expression. For example, if a Boolean function is expressed algebraically as: $F = AB'C$
Then, when A = 1, B = 0, and C = 1, the value of F will be 1. For other values of A, B, and C, the value of F is 0.

Boolean functions can also be represented by truth tables. A *truth table* is the the tabular form of the values of a Boolean function according to all possible values of its variables. For an *n* number of variables, **2n** combinations of **1s and 0s** are listed (from **0** to **2n - 1**), and one column represents function values according to the different combinations. For example, for three variables, the Boolean function **(F = AB + C)** truth table can be written as shown below:
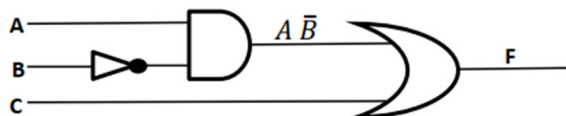
A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	0	0
1	1	1	1
1	1	0	1

For example, the following Boolean functions:-
$$F1 = X\bar{Y}Z \quad \text{and} \quad F2 = \bar{X} + YZ$$
The truth table for the above functions:
The Boolean function can be transformed to logic diagram composed of AND, OR and NOT gates.

Inputs			Outputs	
X	Y	Z	F1	F2
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	
1	1	0	0	
1	1	1	0	

2 Rule of Boolean Algebra

Rule 1: X + 0 = X

X	X+0
0	0+0=0
1	0+1=1

Rule2: X + 1 = 1

X	X +1
0	0 +1=1
1	1+1 =1

Rule3: X + X= X

X	X +X
0	0+0 =0
1	1+1 =1

Rule 4: X + $\bar{X}$= 1

X	**X + $\bar{X}$**
0	0+1=1
1	1+0=1

Rule5: $\bar{\bar{X}}$ = X

X	$\bar{X}$	$\bar{\bar{X}}$
0	1	0
1	0	1

Rule6: X . 1= X

X	X . 1
0	0 .1=0
1	1 .1 =1

Rule7: X . 0 = 0

X	X . 0
0	0 .0=0
1	1 .0=0

Rule8: X . X= X

X	X . X
0	0 .0=0
1	1 .1 =1

Rule9: X . $\bar{X}$= 0

X	X . $\bar{X}$
0	0 .1=0
1	1 .0 =0

Rule10: X$\oplus$ 0= X

X	**X$\oplus$ 0**
0	0 $\oplus$ 0=0
1	1 $\oplus$ 0 =1

X	X $\oplus$ 1
0	0 $\oplus$ 1=1
1	1 $\oplus$ 1 =0

Rule11: X $\oplus$ 1= $\bar{X}$

X	X $\oplus$ X
0	0 $\oplus$ 0=0
1	1 $\oplus$ 1=0

Rule12: X $\oplus$ X= 0

X	X $\oplus$ $\bar{X}$
0	0 $\oplus$ 1=1
1	1 $\oplus$ 0=1

Rule13: X $\oplus$ $\bar{X}$= 1

Prove that X+X=X

$$X + X = (X+X).1$$
$$= (X+X).(X + \bar{X}) \qquad \text{rule 4}$$
$$= X + X \bar{X}$$
$$= X + 0 \qquad\qquad \text{rule 9}$$
$$= X$$

3 Laws of Boolean Algebra

The manipulation of algebraic expressions is based on fundamental laws. Some of these laws extend to the manipulation of Boolean expressions. For example, the commutative law of algebra which states that the result of an operation is the same regardless of the order of operands holds true for Boolean algebra too.

Commutative law	X + Y= Y + X	X.Y= Y.X
Associative law	(X+Y)+Z= Y+(X+Z)	(X.Y).Z= Y.(X.Z)
Distributive law	X(Y+Z)=X.Y+X.Z	X+YZ=(X+Y).(X+Z)

Many students of algebra are frustrated by problems requiring simplification. Sometimes it feels as if extrasensory perception is required to see where the best path to simplification lies. Unfortunately, Boolean algebra is no exception. There is no substitute for practice. Therefore, this section provides a number of examples of simplification in the hope that seeing them presented in detail will give you the tools you need to simplify the problems on your own.

1) $X + 0 = X$
2) $X + 1 = 1$
3) $X + X= X$

9) $X . \bar{X}= 0$
10) $X \oplus 0= X$
11) $X \oplus 1= \bar{X}$

4) $X + \bar{X} = 1$

5) $\bar{\bar{X}} = X$

6) $X . 1 = X$

7) $X . 0 = 0$

8) $X . X = X$

12) $X \oplus X = 0$

13) $X \oplus \bar{X} = 1$

14) $X + XY = X$

15) $X . \bar{X}Y = X + Y$

16) $(X+Y).(X+Z) = X + YZ$

Boolean Expression of three-variable of the XOR

$$A \oplus B \oplus C = (A\bar{B} + \bar{A}B)\bar{C} + \overline{(A\bar{B} + \bar{A}B)}C$$
$$= A\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}BC$$

Exclusive – OR Gate

$Z = X \oplus Y$
$= \bar{X} Y + X \bar{Y}$

The truth table for $A \oplus B \oplus C$

A	B	C	A⊕B	(A⊕B)⊕C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

Prove that: $\bar{A} + AB = \bar{A} + B$

$\bar{A} + AB = \bar{A}(1 + B) + AB$

$\quad\quad = \bar{A} + \bar{A}B + AB$

$\quad\quad = \bar{A} + B(\bar{A} + A)$

$\quad\quad = \bar{A} + B$

Prove that: $A + \bar{A}B = A + B$

$A + \bar{A}B = A(1 + B) + \bar{A}B$

$\quad\quad = A + AB + \bar{A}B$

$\quad\quad = A + B(A + \bar{A})$

$\quad\quad = A + B$

Example: Prove that A + A·B = A
Solution

$$A + A·B = A·1 + A·B \qquad \text{Rule: } A · 1$$
$$= A·(1 + B) \qquad \text{Distributive Law}$$
$$= A·(B + 1) \qquad \text{Commutative Law}$$
$$= A·1 \qquad \text{Rule: } A + 1 = 1$$
$$= A \qquad \text{Rule: } A · 1 = A$$

Remember also that rules of Boolean algebra can be proven using a truth table. The example below uses a truth table to derive another rule.

Example: Prove $A + \overline{A}. B = A + B$
The truth table below goes step-by-step through both sides of the expression to prove.

A	B	$\overline{A}$	$\overline{A}.B$	$A+\overline{A}.B$	A+B
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

Example: Prove (A + B)·(A + C) = A + B·C
Solution

$$(A + B)·(A + C) = (A + B)·A + (A + B)·C \qquad \text{Distributive Law}$$
$$= A·A + B·A + A·C + B·C \qquad \text{Distributive Law}$$
$$= A + B·A + A·C + B·C \qquad \text{Rule: } A·A = A$$
$$= A + A·B + A·C + B·C \qquad \text{Commutative Law}$$
$$= A + A·C + B·C \qquad \text{Rule: } A + A·B = A$$
$$= A + B·C \qquad \text{Rule: } A + A·B = A$$

Now that you have a taste for the manipulation of Boolean expressions, the next section will show examples of how complex expressions can be simplified.

Example: prove $AB + \overline{A}C + BC = AB + \overline{A} C$
Solution

$$= AB + \overline{A} C + 1 · BC \qquad \text{rule } 1.X=X$$
$$= AB + \overline{A} C + (A + \overline{A}) · BC \qquad \text{rule } X+\overline{X}=1$$
$$= AB + \overline{A} C + ABC + \overline{A}BC \qquad \text{Distributive Law}$$
$$= AB + ABC + \overline{A} C + \overline{A}BC \qquad \text{Commutative Law}$$
$$= AB(1+C) + \overline{A} C(1+B) \qquad \text{Distributive Law}$$
$$= AB + \overline{A} C \qquad \text{rule } 1+X=1$$

Example: Show that X+ Y + $\bar{X}.\bar{Y}$ = 1

X+ Y + $\bar{X}.\bar{Y}.$	= (X+Y+$\bar{X}$) (X+Y+$\bar{Y}$)	(Distributive Law) X + YZ = (X + Y)(X + Z)
	=(X+$\bar{X}$ + Y) (X+Y+$\bar{Y}$)	(Commutative Law) X + Y = Y + X
	=(1+Y)(X+1)	Rule 4
	=1.1	Rule2
	=1	Rule8

Example: Show (X+ Y). $\bar{X}.\bar{Y}$ =0

(X+ Y). $\bar{X}.\bar{Y}$	= (X.$\bar{X}.\bar{Y}$ + Y.$\bar{X}.\bar{Y}$)	(Distributive Law) X + YZ = (X + Y)(X + Z)
	=(X.$\bar{X}.\bar{Y}$ + Y.$\bar{Y}.\bar{X}$)	(Commutative Law) X + Y = Y + X
	=(0.$\bar{Y}$ + 0.$\bar{X}$)	
	= (0 + 0)	
	= 0	

Example: determine the truth table and logic diagram AB+ A (B+C)+ B(B+C)

$$AB+ A (B+C)+ B(B+C) = AB+AB+AC+BB+BC$$
$$=AB+AB+AC+B+BC$$
$$= AB+AC+B(1+C)$$
$$= AB+B + AC$$
$$=B(A+1)+AC$$
$$= B+AC$$

(a)   These two circuits are equivalent   (b)
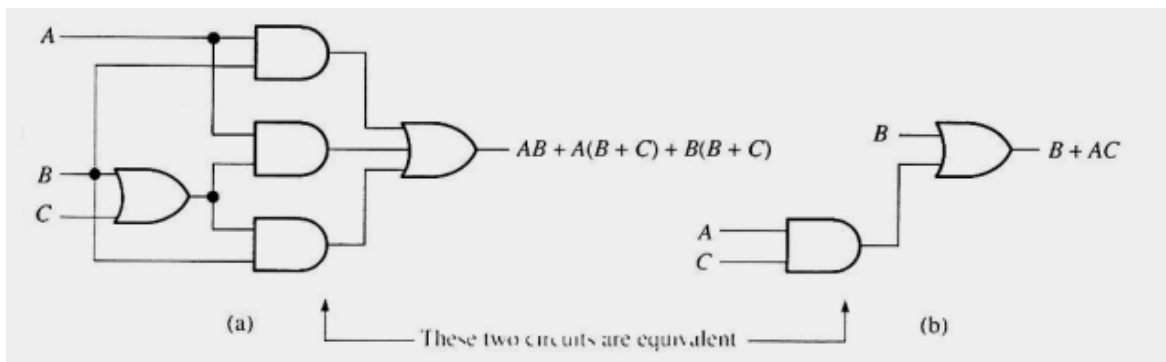
4 Simplification

When a Boolean expression is implemented with logic gates, each literal in the function is designated as input to the gate. The literal may be a primed or unprimed variable. ***Minimization of the number of literals and the number of terms leads to less complex circuits as well as less number of gates***, which should be a designer's aim. There are **several** methods to minimize the Boolean function. In this lecture, **simplification** or minimization of complex algebraic expressions will be shown with the *help of postulates and theorems of Boolean algebra*.

Example: Simplify F= (A.B+C)(A.B+D)

From the rules of Boolean algebra, we know that $(A + B)(A + C) = A + BC$. Substitute $A \cdot B$ for A, C for B, and D for C and we get: $(A \cdot B + C)(A \cdot B + D) = A \cdot B + C \cdot D$

Example: Simplify F= $\overline{Y}.(X+\overline{X}.Y)$

$\qquad = \overline{Y}.X + \overline{Y}.\overline{X}.Y$ Distributive Law

$\qquad = \overline{Y}.X + \overline{X}.\overline{Y}.Y$ Associative Law

$\qquad = \overline{Y}.X + \overline{X}.0$ Anything AND'ed with its inverse is 0

$\qquad = \overline{Y}.X$

$\qquad = X.\overline{Y}$

Example: *Simplify the Boolean function F=AB+ BC + B'C.*

Solution. F = AB + BC + B′C

$\qquad = AB + C (B + B′)$

$\qquad = AB + C$

Example: *Simplify the Boolean function F= A + A'B.*

Solution. F = A+ A′B

$\qquad = (A + A′) (A + B)$

$\qquad = A + B$

Example *: Simplify the Boolean function F= A'B'C + A'BC + AB'.*

Solution. F = A′B′C + A′BC + AB′

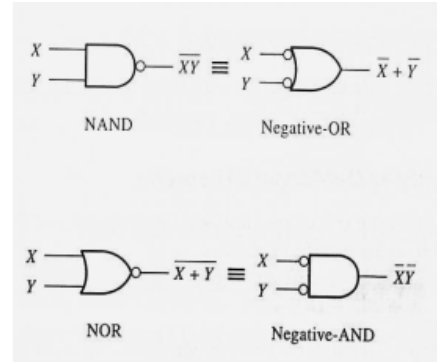$\qquad = A′C (B′+B) + AB′$

$\qquad = A′C + AB′$

H.W:*Use Boolean algebra to simplify the following Boolean function*s:

*1. F = AB + (AC)' + AB'C(AB + C) (Ans: **F=1**)*

*2. F = ((XY′ + XYZ)' + X(Y + XY′))' (Ans: **F=0**)*

*3. F = XYZ + XY'Z + XYZ' (Ans: **F=X(Y+Z)**)*

4. F = $(\overline{X}+Y).(X+\overline{Y})$s

5. $F = \overline{X}.\overline{Y}.\overline{Z} + \overline{X}.\overline{Y}.Z + \overline{X}.Y.\overline{Z} + \overline{X}.Y.Z$

5 Demorgan's Theorem

The complement of the function F is $\bar{F}$ and is obtained from an interchange of 0's for 1's and 1's for 0's in the value of F. the complement of a function may be derived algebraically through DeMorgan's theorem . There are two theorem of DeMorgan:

$$\overline{X.Y} = \bar{X} + \bar{Y}$$

X	Y	$\overline{X.Y}$	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$$\overline{X + Y} = \bar{X}.\bar{Y}$$

X	Y	$\overline{X + Y}$	$\bar{X}.\bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NAND   Negative-OR

NOR   Negative-AND

Complementing Functions:

Use DeMorgan's Theorem to complement a function:

1. Interchange AND and OR operators
2. Complement each constant value and literal

Ex: Fined the complement of the following functions?

$$1 - F1 = \overline{X}\,Y\,\overline{Z} + \overline{X}\,\overline{Y}\,Z \qquad\qquad 2 - F2 = X\,(\,\overline{Y}\,\overline{Z} + Y\,Z\,)$$

Canonical and Standard Forms

Any Boolean function can be expressed in a canonical form, canonical form includes:-

1- Sum of minterms (SOM) also known as *Sum of products* (**SOP**).
2- Product of Maxterms (POM) also known as *Product of sum* (**POS**).

Product Term: An **AND** function is referred to as a product term. The variables in a product term can be either in **true** form or in **complemented** form. For example, **ABC′** is a product term.

Sum Term: An **OR** function is referred to as a sum term. Variables in a sum term can also be either in **true** form or in **complemented** form. For example:
A + B + C′ is a sum term.

Example: Express the following complement of product as sum of complement using Demorgan's theorem:

1- $\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

2- $\overline{ABCD} = \bar{A} + \bar{B} + \bar{C} + \bar{D}$

3- $\overline{ABCEF} = \bar{A} + \bar{B} + \bar{C} + \bar{D} + \bar{E} + \bar{F}$

4- $\overline{A + B + C} = \bar{A}\bar{B}\bar{C}$

5- $\overline{A + B + C + D} = \bar{A}\bar{B}\bar{C}\bar{D}$

6- $\overline{A + B + C + E + F} = \bar{A}\bar{B}\bar{C}\bar{D}\bar{E}\bar{F}$

Example: Prove that $(\overline{X + Y})Z + X\bar{Y} = \bar{Y}(X + Z)$

$= \bar{X}.\bar{Y}.Z + X.\bar{Y}$ (DeMorgan's Law)

$= \bar{Y}(\bar{X}.Z + X)$ (Distributive Law)

$= \bar{Y}(\bar{X} + X)(Z + X)$ (Distributive Law)

$= \bar{Y}.1(Z + X)$ Rule 4

$= \bar{Y}(Z + X)$

Example: Simplify the following Boolean Expression $\overline{AB + AC} + \overline{A}BC$

Step1: apply DeMoragn's theorem to the first term.

$(\overline{AB}).(\overline{AC}) + \overline{A}BC$

Step2: apply DeMoragn's theorem to each term in parentheses.

$(\bar{A} + \bar{B})(\bar{A} + \bar{C}) + \overline{A}BC$

Step3: Apply the distributive law to the two terms in parentheses.

$\bar{A}\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \overline{A}BC$

Step4:Apply Boolean Rules $\bar{A}\bar{A} = \bar{A}$, $\overline{A}\overline{B} + \overline{A}BC = \overline{AB}(1 + C) = \overline{A}\overline{B}$

$\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C}$

Step5: Apply Boolean Rule $\bar{A} + \bar{A}\bar{C} = \bar{A}(1 + \bar{C}) = \bar{A}$ to the first and second terms

$\bar{A} + \bar{A}\bar{B} + \bar{B}\bar{C}$

Step6: Apply Boolean Rule $\bar{A} + \bar{A}\bar{B} = \bar{A}(1 + \bar{B}) = \bar{A}$ to the first and second terms

$\bar{A} + \bar{B}\bar{C}$

## 6 Sum of Product (SOP) and Product of Sum (POS)

A **sum-of-products (SOP)** expression is a Boolean expression in a specific format. The term sum-of-products comes from the expression's form: a sum (OR) of one or more products (AND). It is basically an *OR operation on AND operated variables.* For example, F = AB + BC + AC or F = A′B + BC + AC′ are sum of products expressions. As a digital circuit, an SOP expression takes the output of one or more AND gates and OR's them together to create the final output. The inputs to the AND gates are either

inverted or non-inverted input signals. This **limits the number of gates that any input signal passes** through before reaching the output to an inverter, an AND gate, and an OR gate.

$$\overline{A}BC\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$$

This also means that an SOP expression cannot have more than one variable combined in a term with an inversion bar.

Example for Sum of Product (SOP): X=AB+BCD+AC
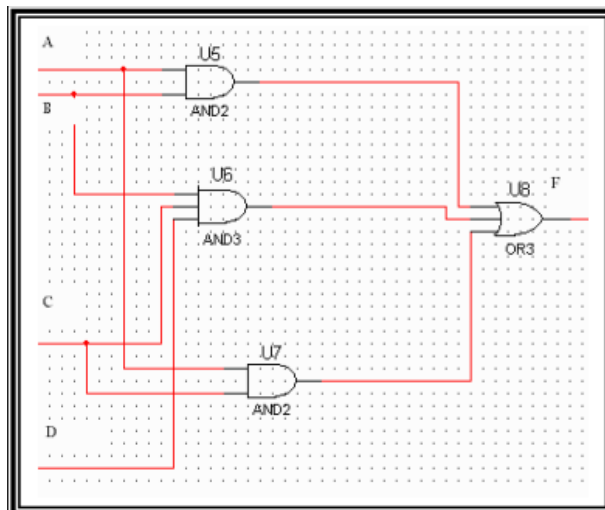
Examples:

a- **AB+B(CD+EF)=AB+BCD+BEF**

b- **(A+B)(B+C+D)=AB+AC+AD+BB+BC+BD**

c- $\overline{\overline{(A+B)}+C}=\overline{\overline{(A+B)}*\overline{C}} =(A+B)\overline{C}=A\overline{C}+B\overline{C}$

Example Sum of product

$$F=\overline{A}B+A\overline{B}$$

A	B	X	F
0	0	0	
0	1	1	$\overline{A}B$
1	0	1	$A\overline{B}$
1	1	0	

The product-of-sums (POS) format of a Boolean expression is much like the SOP format with its two levels of logic (not counting inverters). The difference is that the outputs of multiple OR gates are combined with a single AND gate which outputs the final result. It is an AND operation on OR operated variables. For example, **F = (A + B + C) (A + B′ + C) (A + B + C′)** or **F = (A + B + C) (A′ + B′ + C′)** are product of sums
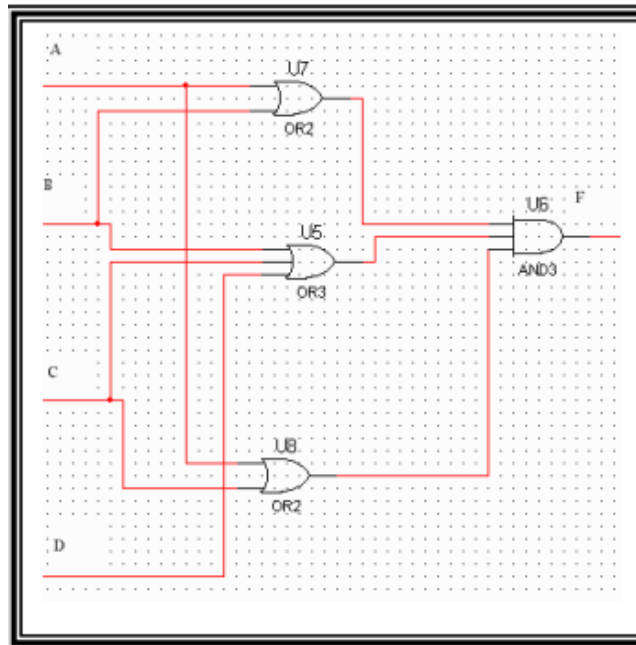
expressions. The expression below adheres to the format of a POS expression.

$(\overline{A}+B+C+\overline{D})(A+\overline{B}+D)(C+\overline{D})(A+\overline{D})$

As with SOP expressions, a POS expression cannot have more than one variable combined in a term with an inversion bar. For example, the following is not a POS expression:

$$(\overline{A + B + C}+D)(A+\overline{B}+D)(C+\overline{D})(A+\overline{D})$$

Example of Product of Sum (POS): (A+B)(B+C+D)(A+C)

Example:

$F=(\overline{A}+\overline{B})(A+B)$

A	B	X	F
0	0	0	$\overline{A}+\overline{B}$
0	1	1	
1	0	1	
1	1	0	A+B

Standard form: In this form, the terms form the function may contain one, two, or any number of literals. There are two types of standard form: sum of the products (SOP) or product of the sums (POS) fashion. For examples; F = AB + BC + AC or F = (A + B + C) (A + B′ + C) (A + B + C′) are the standard forms.

However, Boolean functions are also sometimes expressed in nonstandard forms like F = (AB + CD) (A′B′ + C′D′), which is neither a sum of products form nor a product

of sums form. However, the same expression can be converted to a standard form with help of various Boolean properties, as

F = (AB + CD) (A′B′ + C′D′) = A′B′CD + ABC′D′

The following is not an SOP expression:

$$(\overline{AB})C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$$

This is because the first term has A and B passing through a NAND gate before being AND'ed with C and D thereby creating a third level of logic. To fix this problem, we need to break up the NAND using DeMorgan's Theorem.

$$(\overline{AB})C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$$
$$(\overline{A}+\overline{B})C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$$
$$\overline{A}C\overline{D}+\overline{B}C\overline{D} + A\overline{B}D + C\overline{D} + A\overline{D}$$

7 Minterms and Maxterms for three Binary variables A Product term which contains each of variables as factor in either complemented or uncomplemented forms is called a minterm. A sum which contains each of n variables complemented or not is called maxterm.

In other words, A *product* term containing all *n-variables* of the function in either *true (1)* or *complemented (0)* form is called the **Minterm**. In **Minterm**, we look for the functions where the output results in "**1**" **(F=0)**.

A *sum* term containing all *n variables* of the function in either *true* or *complemented* form is called *the Maxterm*. In **Maxterm** we look for function where the output results in "**0**" **(F=0)**.

Truth table representing Minterm and Maxterm

X	Y	Z	Minterms Product Terms	Maxterms Sum Terms
0	0	0	$m_0 = \overline{X}\cdot\overline{Y}\cdot\overline{Z} = \min(\overline{X},\overline{Y},\overline{Z})$	$M_0 = X+Y+Z = \max(X,Y,Z)$
0	0	1	$m_1 = \overline{X}\cdot\overline{Y}\cdot Z = \min(\overline{X},\overline{Y},Z)$	$M_1 = X+Y+\overline{Z} = \max(X,Y,\overline{Z})$
0	1	0	$m_2 = \overline{X}\cdot Y\cdot\overline{Z} = \min(\overline{X},Y,\overline{Z})$	$M_2 = X+\overline{Y}+Z = \max(X,\overline{Y},Z)$
0	1	1	$m_3 = \overline{X}\cdot Y\cdot Z = \min(\overline{X},Y,Z)$	$M_3 = X+\overline{Y}+\overline{Z} = \max(X,\overline{Y},\overline{Z})$
1	0	0	$m_4 = X\cdot\overline{Y}\cdot\overline{Z} = \min(X,\overline{Y},\overline{Z})$	$M_4 = \overline{X}+Y+Z = \max(\overline{X},Y,Z)$
1	0	1	$m_5 = X\cdot\overline{Y}\cdot Z = \min(X,\overline{Y},Z)$	$M_5 = \overline{X}+Y+\overline{Z} = \max(\overline{X},Y,\overline{Z})$
1	1	0	$m_6 = X\cdot Y\cdot\overline{Z} = \min(X,Y,\overline{Z})$	$M_6 = \overline{X}+\overline{Y}+Z = \max(\overline{X},\overline{Y},Z)$
1	1	1	$m_7 = X\cdot Y\cdot Z = \min(X,Y,Z)$	$M_7 = \overline{X}+\overline{Y}+\overline{Z} = \max(\overline{X},\overline{Y},\overline{Z})$

Ex Find the 1 – Sum of Minterms 2 – Product of Maxterms of **F**?

1 – Sum of minterms:

$$F = (\overline{X}\overline{Y}Z) + (X\overline{Y}\overline{Z}) + (XYZ)$$

$$= m1 + m4 + m7 = \sum(1, 4, 7)$$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

2- Product of Maxterms:

$$F = (X + Y + Z) \cdot (X + \overline{Y} + Z) \cdot (X + \overline{Y} + \overline{Z}) \cdot (\overline{X} + Y + \overline{Z}) \cdot (\overline{X} + \overline{Y} + Z)$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6 = \prod(0, 2, 3, 5, 6)$$

Ex Express each of the following functions in Canonical Forms

X	Y	Z	F1	F2
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

Ex Express the Boolean function $F = A + \overline{B}C$ in **Sum of minterms**?

Sol:

The function has three variables A, B and C. The first term **missing two** variables

$$A = A \cdot 1 \qquad\qquad \text{by } X \cdot 1 = X$$

$$= A(B + \overline{B}) \qquad\quad \text{by } X + \overline{X} = 1$$

$$= AB + A\overline{B}$$

$$= AB(C + \overline{C}) + A\overline{B}(C + \overline{C}) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C}$$

The second term **missing one** variable

$$\overline{B}C = \overline{B}C(A + \overline{A}) = A\overline{B}C + \overline{A}\,\overline{B}C$$

$$F(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$
$$= 111 + 110 + 101 + 100 + 001$$

$$= m7 + m6 + m5 + m4 + m1 = \Sigma(1, 4, 5, 6, 7)$$

Ex Express the Boolean function $F = X\,Y + \overline{X}\,Z$ in **product Maxterms**

First the function must converted into OR terms using distributed low

$$F = XY + \bar{X}Z = (XY + \bar{X}) \cdot (XY + Z)$$
$$= (X + \bar{X}) \cdot (Y + \bar{X}) \cdot (X + Z) \cdot (Y + Z)$$

$$= (\bar{X} + Y) \cdot (X + Z) \cdot (Y + Z)$$

Each term is missing one variable

$$\bar{X} + Y = \bar{X} + Y + 0 \quad by\ X + 0 = X$$

$$= \bar{X} + Y + Z\bar{Z} \qquad by\ X \cdot \bar{X} = 0$$

$$= (\bar{X} + Y + Z) \cdot (\bar{X} + Y + \bar{Z}) \quad by\ distributive\ law$$

$$X + Z = X + Z + 0 = X + Z + Y \cdot \bar{Y} = (X + Y + Z)(X + \bar{Y} + Z)$$

$$Y + Z = Y + Z + 0 = Y + Z + X \cdot \bar{X} = (X + Y + Z)(\bar{X} + Y + Z)$$

$$F = (\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})(X + Y + Z)(X + \bar{Y} + Z)$$

$$= M_4 \cdot M_5 \cdot M_0 \cdot M_2 = \prod(0, 2, 4, 5)$$

Ex Express the Boolean function F in 1- Sum of minterms 2 – Product of Maxterms

$$F = \overline{A}(B + \overline{C})$$

Solution: 1. Sum of Minterms

$$F(A, B, C) = \overline{A}(B + \overline{C}) = \overline{A}B + \overline{A}\,\overline{C}$$

The first term missing one variable

$$\overline{A}B = \overline{A}B . 1 = \overline{A}B(C + \overline{C}) = \overline{A}BC + \overline{A}B\overline{C}$$

The second term missing one variable

$$\overline{A}\,\overline{C} = \overline{A}\,\overline{C}.1 = \overline{A}\,\overline{C}(B + \overline{B}) = \overline{A}B\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$$

$$F(A,B,C) = \overline{A}BC + \overline{A}B\overline{C} + \overline{A}\,\overline{B}\,\overline{C} = 011 + 010 + 000$$

$$= m3 + m2 + m0 = \Sigma(0,2,3)$$

2. Product of Maxterms

$$F(A,B,C) = \overline{A}(B + \overline{C})$$

The first term missing two variables

$$\overline{A} = \overline{A} + 0 = \overline{A} + B\overline{B} = (\overline{A} + B)(\overline{A} + \overline{B})$$
$$= (\overline{A} + B + 0)(\overline{A} + \overline{B} + 0)$$
$$= (\overline{A} + B + C\overline{C})(\overline{A} + \overline{B} + C\overline{C})$$
$$= (\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$$

The second term missing one variable

$$B + \overline{C} = B + \overline{C} + 0 = B + \overline{C} + A\overline{A} = (B + \overline{C} + A)(B + \overline{C} + \overline{A})$$
$$= (A + B + \overline{C})(\overline{A} + B + \overline{C})$$

$$F(A,B,C) = (\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})(A + B + \overline{C})$$

$$= M4 . M5 . M6 . M7 . M1 = \Pi(1,4,5,6,7)$$

Ex: Express the function F in Sum of minterms and Product of Maxterms and simplify it

1– Sum of minterms

$$F(X,Y,Z) = \overline{X}\,Y\overline{Z} + \overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + XYZ$$

$$= \overline{X}Y(\overline{Z} + Z) + X\overline{Y}Z + XY(\overline{Z} + Z)$$

$$= \overline{X}Y + X\overline{Y}Z + XY = Y(\overline{X}+X) + X\overline{Y}Z = Y + X\overline{Y}Z$$

$$= X\overline{Y}Z + Y = (XZ + Y)(\overline{Y} + Y) = XZ + Y$$

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

H.W.

1. Obtain the canonical product of the sum form of the following function.

$$F\ (A,\ B,\ C) = A + B'C$$

2. Obtain the canonical product of the sum form of the following function.

$$F\ (A,\ B,\ C) = (A + B')\ (B + C)\ (A + C')$$

3. Obtain the canonical sum of product form of the following function.

$$F\ (A,\ B) = A + B$$

4. Prove the following using Boolean theorems:

(a) $(A + C)(A + D)(B + C)(B + D) = AB + CD$
(b) $(X + Y) \oplus (X + Z) = X'\ (Y \oplus Z)$.

5. Find the Boolean expression for F, when F is 1 only if A is 1 and B is 1, or if A is 0 and B is 0.

6. Convert $F = ABCD + A'BC + B'C'$ into a sum of minterms by algebraic method.

7. Convert $F = AB + B'CD$ into a product of maxterms by algebraic method.