The Karnaugh Map

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Karnaugh Maps are graphical representations of truth tables. It is an alternative algebraic expressions for the same function are derived by recognizing patterns of squares. It consist of a grid with one cell for each row of the truth table. The intersection of each row and column corresponds to a unique set of input values. The purpose of Karnaugh maps is to rearrange truth tables so that adjacent cells can be represented with a single product using the simplification previously described. Each square represents a minterm. Adjacent squares differ in the value of one variable. Alternative algebraic expressions for the same function are derived by recognizing patterns of squares

1-Two- Variable Karnaugh map

X	Y	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

X \ Y	Y=0	Y=1
X=0	m_0=XY	$m_1=\bar{X}Y$
X=1	m_2=XY	m_3=XY

For example the truth table for OR gate

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

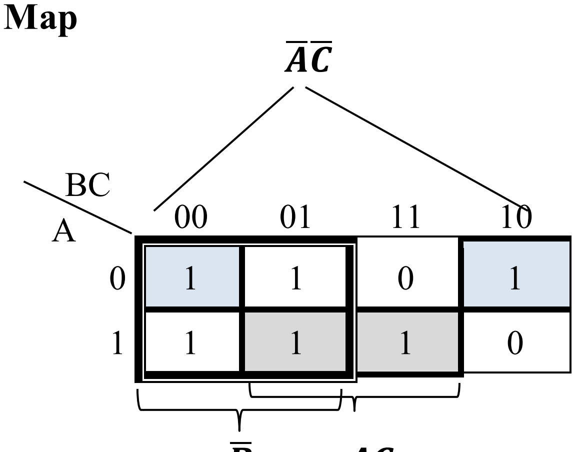
X \ Y	0	1
0	0	1
1	1	1

F= X+Y

Two pairs of adjacent cells containing 1's can be combined using the Minimization.

2-Three- Variable Karnaugh Map

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0



1	1	1	1

F(A,B,C)=∑(0,1,2,4,5,7)=$\overline{B} + \overline{A}\overline{C} + AC$

Note that: One square represents a minterm with three variables. Two adjacent squares represent a product term with two variables. Four "adjacent" terms represent a product term with one variable. Eight "adjacent" terms is the function of all ones (no variables) = 1.

<u>Example:</u> simplify the Boolean Function F(X,Y,Z)= ∑(0,2,4,6)

Solution: since the function has three variables, a three variable map must be used

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

X \ YZ	00	01	11	10
0	1	0	0	1
1	1	0	0	1

F=$\overline{Z}$

<u>Example:</u> for the given Boolean function F=$\overline{A}\overline{B} + BC + A\overline{C} + ABC$

Express it in sum of minterms and find the minimal sum of products

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	1	1	1	0
1	0	1	1	0

F=$\overline{A}\overline{B} + BC + A\overline{C} + ABC$

=C+$\overline{A}\overline{B}$

F(A,B,C)= ∑(0,1,3,5,7)

3-Four-Variable Karnaugh Map

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Note: Four variable maps can have rectangles corresponding to:

- A single 1 = 4 variables, (i.e. Minterm)
- Two 1s = 3 variables,
- Four 1s = 2 variables
- Eight 1s = 1 variable,
- Sixteen 1s = zero variables (i.e. Constant "1")

Example: Find a simplified expression for the function whose truth table is:

A	B	C	D	F
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**1**

AB \ CD	00	01	11	10
00	1	1	0	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

$$F=\bar{C}+A\bar{B}+BD+\bar{A}\bar{D}$$

Example: Find a simplified expression for the function whose truth table is:

A	B	C	D	F1	F2	F3
0	0	0	0	0	1	0
0	0	0	1	0	0	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	1	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	1
0	1	1	1	1	1	0
1	0	0	0	0	1	0

1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	1	0	1
1	1	0	0	0	0	0
1	1	0	1	0	1	0
1	1	1	0	0	0	0
1	1	1	1	1	0	1

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	0	0	1	0
10	0	0	1	1

$F1=\bar{A}B\bar{C}+BCD+A\bar{B}C$

AB \ CD	00	01	11	10
00	1	0	1	1
01	0	0	1	1
11	0	1	0	0
10	1	0	0	1

$F2=\bar{A}C+\bar{B}\bar{D}+AB\bar{C}D$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	1	0
10	0	0	1	0

$F3=ACD+\bar{A}B\bar{D}$

<u>Example:</u> Simplify the Boolean function: $F(W,X,Y,Z)=\sum(0,4,5,6,8,10,12,13,14)$

	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	1	1	0	1

Solution:

10	1	0	0	1

YZ
WX

$$\mathbf{F = \overline{Y}\overline{Z} + W\overline{Z} + X\overline{Z} + X\overline{Y}}$$

Example: Minimize the following expression: $\mathbf{F = A\overline{B} + \overline{A}C\overline{D} + \overline{C}D + \overline{C} + \overline{B}C}$

$$\mathbf{F = \overline{C} + \overline{B} + \overline{A}\overline{D}}$$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	1	0	0
10	1	1	1	1

"Don't Care" Conditions in a Karnaugh Map

In digital systems it often happens that some input conditions (i.e. some input valuations) can never happen, an input combination that can never happen is referred to as a don't care condition. A don't care condition can be ignored (i.e. the output for that condition can be treated as 0 or 1 in the truth table). A function that has don't care condition(s) is said to be incompletely specified.

For example, the four-input Karnaugh map shown below contains two "don't care" represented in letter (x)

$$\mathbf{F = \overline{A}B + \overline{C}\overline{D} + \overline{B}\overline{D}}$$

AB \ CD	00	01	11	10
00	1	x	0	1
01	x	1	x	x
11	1	0	x	0
10	1	0	0	1

4-Five-Variable Karnaugh Map

Maps for more than four variables are not as simple to use. A five variable map needs 32 squares. The five variable map consists of 2 four variable maps with variables A,B,C,D ,E. Variable A distinguished between the two maps as indicated in the map below.

A=0 **A=1**

DE
BE

DE
BC

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

Example: **Simplify the Boolean function**
F(A,B,C,D,E)=∑(6,7,10,11,15,23,28,29,30,31)
Solution:

A=0

BC \ DE	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	0
10	0	0	1	1

A=1

BC \ DE	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	0	0	0

$$\mathbf{F = ABC + CDE + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D}$$

Example: **Minimize the following expression:** $\mathbf{F = ABC + CDE + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D}$

A=0

BC \ DE	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	0
10	0	0	1	1

A=1

BC \ DE	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	0	0	0

Example: **Simplify the following boolean function in (a) sum of products and (b) Product of sums. F(A,B,C,D)=∑(0,1,2,5,8,9,10), draw the logic Circuit for (a)and (b)**
Solution:
(a) the 1's represented all the minterms of the function

AB \ CD	00	01	11	10

$$\mathbf{F = \overline{B}\overline{C} + \overline{B}\overline{D} + \overline{A}\overline{C}D}$$

00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

(b) if the square marked with 0's are combined , we get the complemented function

$$\overline{F} = AB + CD + BD$$

Applying Demorgan theorem to the complemented Function

$$F = \overline{AB + CD + BD}$$

$$F = (\overline{AB})(\overline{CD})(\overline{BD})$$

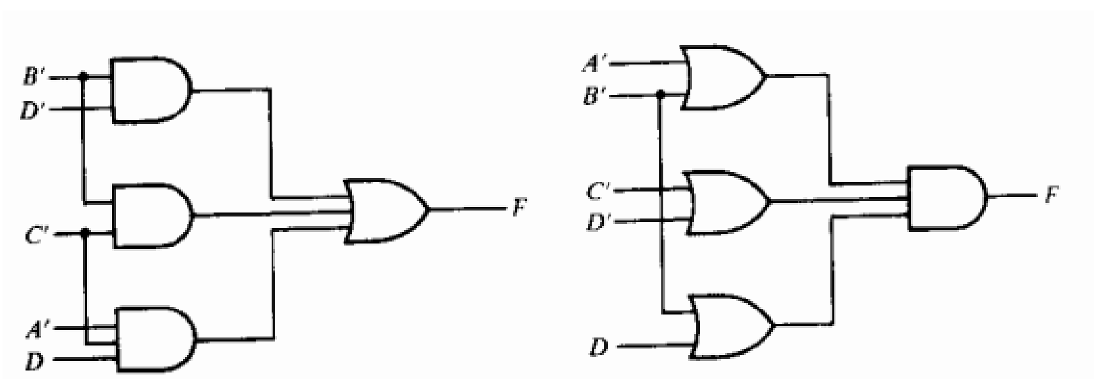
$$F = (\overline{A} + \overline{B})(\overline{C} + \overline{D})(\overline{B} + \overline{D})$$

the 0's represented all the maxterms of the function

$$F = (\overline{A} + \overline{B})(\overline{C} + \overline{D})(\overline{B} + \overline{D})$$

$$F = \prod(3,4,6,7,11,12,13,14,15)$$

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1



(a) $F = B'D' + B'C' + A'C'D$ (b) $F = (A' - B')(C' + D')(B' + D)$

Home Work:

1- Use a K-map to find an optimum SOP equation for $F(X,Y,Z) = \Sigma m(0,1,2,4,6,7)$

2- Use a K-map to find an optimum SOP equation for $F(W,X,Y,Z) = \Sigma m(0,,2,4,5,6,7,8,10,13,15)$

3- Find the optimum POS solution: $F(A,B,C,D) = \Sigma m(3,9,11,12,13,14,15) + \Sigma d\,(1,4,6)$

4-find a simplified expression using K-map for $\mathbf{F = \overline{A}\overline{C}\overline{D} + A\overline{B}D}$, $\mathbf{d = \overline{A}D}$