11 .Numerical analysis and curve fitting

11.5 Characteristic Polynomial and Roots

- Polynomial comes from the Greek *poly*, "many" and medieval Latin *binomium*, "binomial".
- Forming a sum of several terms produces a polynomial. For example, the following is a polynomial:

$$\underbrace{3x^2}_{\text{term}} - \underbrace{5x}_{\text{term}} + \underbrace{4}_{\text{term}}$$

- It consists of three terms: the first is degree two, the second is degree one, and the third is degree zero.

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11.6 Polynomials in MATLAB

- Represented by a row vector in which the elements are the coefficients as

$$[a_n \; a_{n-1} \ldots \; a_2 \;\; a_1 \; a_0]$$

- The a_i elements of this vector are the coefficients of the polynomial in descending order.
- Must include all coefficients, even if 0:

Examples:-

The polynomial

1) $s^3 - 6s^2 - 72s - 27$ is represented in MATLAB software as :

```
>>p = [1 -6 -72 -27]
```

2) 8x + 5 , >>p = [8 5]

3) $6x^2 - 150$, >>h = [6 0 -150]

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11.7 Roots of Polynomials

- We can find the roots of any polynomial with the roots(p) function where p is a row vector containing the polynomial coefficients in descending order.

Example1:

Find the roots of the polynomial

$$p_1(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$$

Solution:

The roots are found with the following two statements. We have denoted the polynomial as p1,and the roots as roots_ p1.

```
>> p1=[1   -10   35   -50   24]   % Specify the coefficients of p1(x)
P1=
           1   -10   35    -50    24
```

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11.7 Roots of Polynomials

```
>> roots_p1=roots(p1)  % Find the roots of p1(x)
roots_p1 =
            4.0000
            3.0000
            2.0000
            1.0000
```

We observe that MATLAB displays the polynomial coefficients as a row vector, and the roots as a column vector.

11 .Numerical analysis and curve fitting

11.7 Roots of Polynomials

Example2:

Find the roots of the polynomial

$$p_2(x) = x^5 - 7x^4 + 16x^2 - 25x + 52$$

Solution:

There is no cube term; therefore, we must enter zero as its coefficient. The roots are found with the statements below where we have defined the polynomial as p2, and the roots of this polynomial as roots_ p2.

```
>> p2=[1   -7   0   16  25   52]
P2=
          1    -7    0    16    25    52
>> roots_ p2=roots(p2)
roots_ p2 =
```

11 .Numerical analysis and curve fitting

11.7 Roots of Polynomials

```
>> roots_ p2=roots(p2)
roots_ p2 =
        6.5014
        2.7428
        -1.5711
        -0.3366 + 1.3202i
        -0.3366 - 1.3202i
```

The result indicates that this polynomial has three real roots, and two complex roots.

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11.8 Polynomial Construction from Known Roots

We can compute the coefficients of a polynomial from a given set of roots with the poly(r) function where r is a row vector containing the roots.

Example3:

It is known that the roots of a polynomial are 1,2,3 and 4. Compute the coefficients of this polynomial.

Solution:

We first define a row vector, say r3, with the given roots as elements of this vector; then, we find the coefficients with the poly(r) function as shown below.

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11.8 Polynomial Construction from Known Roots

```
>>r3=[1 2 3 4] % Specify the roots of the polynomial
r3 =
     1    2    3    4
>>poly_r3=poly(r3) % Find the polynomial coefficients
poly_r3 =
     1   -10   35   -50   24
```

We observe that these are the coefficients of the polynomial p1(x) of Example1.

11 .Numerical analysis and curve fitting

11.8 Polynomial Construction from Known Roots

Example4:

It is known that the roots of a polynomial are −1, −2, −3, 4 + j5, and 4 − j5.Find the coefficients of this polynomial.

Solution:

We form a row vector, say r4, with the given roots, and we find the polynomial coefficients with the poly(r) function as shown below.

```
>> r4=[ -1 -2 -3 4+5j 4-5j ]
```

11 .Numerical analysis and curve fitting

11.8 Polynomial Construction from Known Roots

Example4:

```
>> r4=[ -1 -2 -3 4+5j 4-5j ]
r4 =
  Columns 1 through 4
  -1.0000 + 0.0000i  -2.0000 + 0.0000i  -3.0000 + 0.0000i   4.0000 + 5.0000i
  Column 5
   4.0000 - 5.0000i
>> poly_r4=poly(r4)
```

11 .Numerical analysis and curve fitting

11.8 Polynomial Construction from Known Roots

Example4:

```
>> poly_r4=poly(r4)
poly_r4 =
     1    -2     4   164   403   246
```

Therefore, the polynomial is

$$P_4(x) = x^5 + 14x^4 + 100x^3 + 340x^2 + 499x + 246$$

11 .Numerical analysis and curve fitting

11.9 Evaluation of a Polynomial at Specified Values

The polyval(p,x) function evaluates a polynomial $P(x)$ at some specified value of the independent variable x .

Example5:

Evaluate the polynomial

$$P_5(x) = x^6 - 3x^5 + 5x^3 - 4x^2 + 3x + 2$$

at x = -3 .

Solution:

```
>>p5=[1 −3 0 5 −4 3 2]; % These are the coefficients
```

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11.9 Evaluation of a Polynomial at Specified Values

$$P_5(x) = x^6 - 3x^5 + 5x^3 - 4x^2 + 3x + 2$$

at x = -3 .

Solution:

```
>>p5=[1 −3 0 5 −4 3 2]; % These are the coefficients
>> val_minus3=polyval(p5, −3) % Evaluate p5 at x=−3.
val_minus3 =
        1280
```

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11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

conv(a,b) – multiplies two polynomials a and b

[q,r]=deconv(c,d) – divides polynomial c by polynomial d and displays the quotient q and remainder r.

polyder(p) – produces the coefficients of the derivative of a polynomial p.

polyint(p) – produces the coefficients of the integral of a polynomial p.

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

Example6: Let $P_1(x) = x^5 - 3x^4 + 5x^2 + 7x + 9$

$P_2(x) = 2x^6 - 8x^4 + 4x^2 + 10x + 12$

Compute the product p1.p2 with the conv(a,b) function.

Solution:

```
>> p1=[1 −3 0 5 7 9];
>> p2=[2 0 −8 0 4 10 12];
>> p1p2=conv(p1,p2)
p1p2 =
        2  -6  -8  34  18  -24  -74  -88  78  166  174  108
```

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

```
>> p1p2=conv(p1,p2)

p1p2 =

        2  -6  -8  34  18  -24  -74  -88  78  166  174  108
```

Therefore,

$$P1\,.\,P2_1 \;=\; 2x^{11} - 6x^{10} - 8x^{9} + 34x^{8} - 18x^{7} - 24x^{6} - 74x^{5} - 88x^{4} + 78x^{3}$$

$$+ 166x^{2} + 174x + 108$$

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

Example7: Let $P_3(x) = x^7 - 3x^5 + 5x^3 + 7x + 9$

$P_4(x) = 2x^6 - 8x^2 + 4x^2 + 10x + 12$

Compute the quotient p3/p4 using the deconv(p,q) function.

Solution:

```
>> p3=[1  0  −3  0  5  7  9]; p4 = [2  −8  0  0  4  10  12]; [q,r] = deconv(p3,p4)
q =
        0.5
r =
        0   4   -3   0   3   2   3
```

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

q =

0.5

r =

0 4 -3 0 3 2 3

Therefore, the quotient q(x) and remainder r(x) are :

q(x)=0.5 $r(x) = 4x^5 - 3x^4 + 3x^2 + 2x + 3$

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

Example8: Let $p_5 = 2x^6 - 8x^4 + 4x^2 + 10x + 12$

Compute the derivative dp_5/dx using the polyder(p) function.

Solution:

```
>> p5=[2  0  −8  0  4  10  12];
>>der_p5=polyder(p5)
der_p5 =
        12    0    -32    0    8    10
```

Therefore,

$$dp_5/dx = 12x^5 - 32x^3 + 8x + 10$$

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

Example9:

Let $p_6 = 6x^2$

Compute the integral $\int p_6 dx$ using the polyint(p) function.

Solution:

```
>> p6=[6  0  0];
>>der_p6=polyint(p5)
int_p6 =
        2    0    0    0
```

Therefore,

$$\int p_6 \, dx = 2x^3$$

11 .Numerical analysis and curve fitting

11.11 Curve fitting

- Matlab also has a convenient tool for curve fitting. If we have two vectors, x and y, with paired observations, we can approximate the functional relation between them with a polynomial of some degree.

- If the degree is 1, the relation is linear;

- if it is 2, the relation is quadratic, etc.

- This can be done with the function polyfit().

11 .Numerical analysis and curve fitting

- The following script estimates the coefficients of polynomials of order 1, 2, and 3, for a given set of observations, and plots the results in three graphs.

```
clc,clear all
x = [1 2 3 4 5 6 7 8 9]; y = [2 3 3 5 7 8 8 9 7];
x_val = linspace(0,10,100);
for degree=1:3
poly = polyfit(x,y,degree);
disp(['Coeff., case ' num2str(degree) ': ' num2str(poly)])
y_val = polyval(poly,x_val);
subplot(3,1,degree)
plot(x,y,'r*'), axis([0 10 0 10])
hold on
plot(x_val,y_val)
ylabel(['Degree: ' num2str(degree)])
end
```

11 .Numerical analysis and curve fitting

11.11 Curve fitting: poly = polyfit(x,y,degree)

The output is:

Coeff., case 1: 0.85 1.5278
Coeff., case 2: -0.12229 2.0729 -0.71429
Coeff., case 3: -0.053872 0.68579 -1.3318 2.8413

- The first two inputs to polyfit() are the vectors of X- and Y-values, and the third is the degree of the polynomial (i.e., the highest value of the exponent).

poly = polyfit(x,y,degree);

- The function responds with a matrix that holds one more element than the degree.
- The elements of the matrix are the coefficients of the estimated polynomial.
- For example, in the third case above

$$y = -0.053872x^3 + 0.68579x^2 - 1.3318x + 2.8413$$

11 .Numerical analysis and curve fitting

11.11 Curve fitting: poly = polyfit(x,y,degree)

- The function polyval() uses a matrix of coefficients, poly above, and returns Y-values for given X-values.

y_val = polyval(poly,x_val);

- Figure 11-1 shows the resulting three plots. The red markers are the same in all three cases, but the curves correspond to the fitted polynomials.

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11.11 Curve fitting: poly = polyfit(x,y,degree)

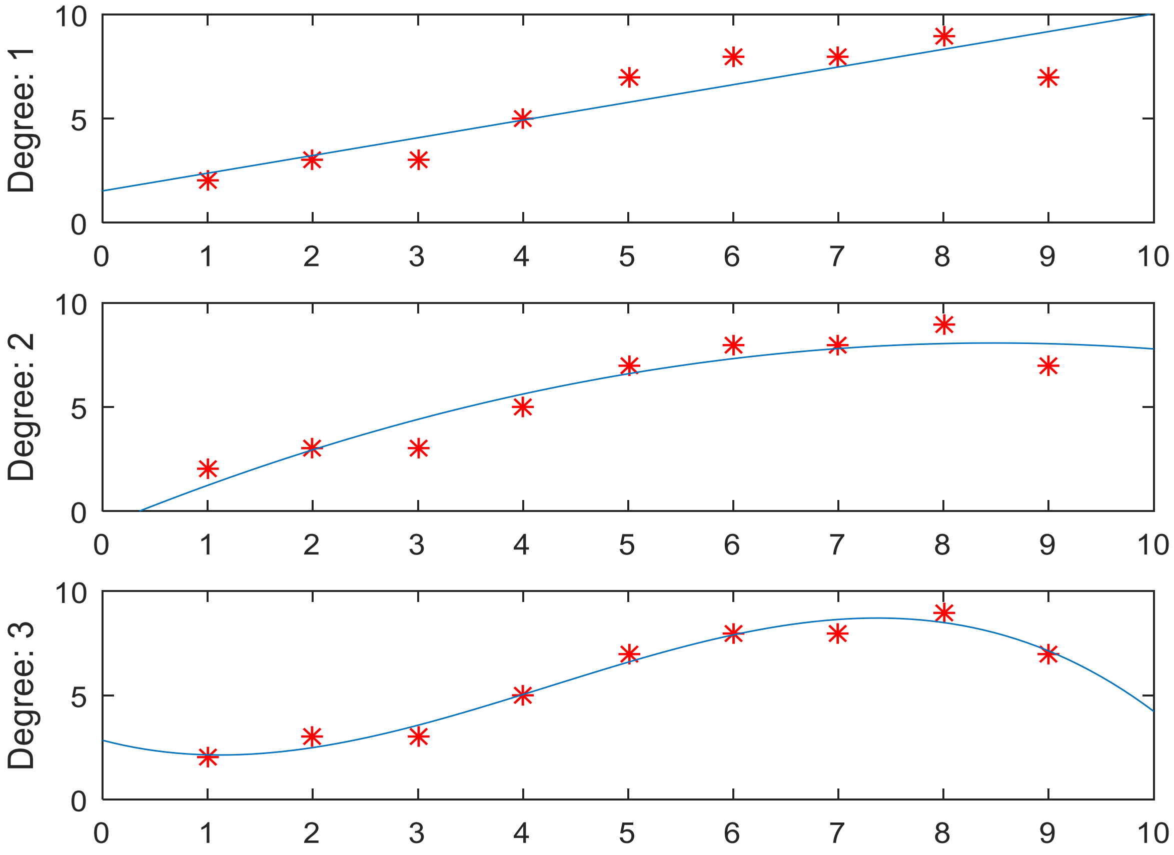

Figure 11-1

Examples of curve fitting