OPERATORS : ^

An operator is a symbol for a mathematical procedure that changes one function into another.

An operator a rule that transforms a given function into another function .

Operators arise because in quantum mechanics you are describing nature with waves (the wave function) rather than with discrete particles whose motion can described with the deterministic equations of Newtonian physics.

Operators are more important in classical mechanics and quantum mechanics .

Why are operators important in quantum mechanics?

Operators used in quantum mechanics can decide the eigenvalues, eigenstates and also make transformation on the quantum states .

Example 1:

$\hat{D}(X^2 + 3e^X)$ if $\hat{D} = 3$

Solution: $3(X^2 + 3e^x)$

$= 3X^2 + 9e^X$

Example 2 :

If $\hat{D} = \partial / \partial x$ ....$(\hat{D} + \hat{3})(X^3 - 5)$

$= (\hat{D} X^3 - \hat{D} 5) + (\hat{3} X^3 - 15)$

$= \partial/\partial x\, X^3 - \partial/\partial x\, 5 + (3X^3 - 15)$

$= 3X^2 + 3X^3 - 15$

$= 3X^3 + 3X^2 - 15$

Types of OPERATORS

1-Commute

P^Q^F(x)= Q^P^F(x)

Example 3

If $P\hat{} = (\partial/\partial x)_{zy}$, $Q\hat{} = (\partial/\partial y)_{zx}$

sol/

P^Q^ F(x) = (∂/∂x) zy (∂/∂y) zx F(x)

$= \partial^2 F(x)/ (\partial x \partial y)_{xyz}$

$Q\hat{}P\hat{}F(x) = (\partial/\partial y)_{zx}(\partial/\partial x)_{zy} F(x)$

$= \partial^2 F(x)/(\partial y \partial x)_{xyz}$

2-non commute operators

P^Q^F(x)≠ Q^P^F(x)

Example 4

If P^= 5+ , Q^= √ , F(x)= 4

P^ Q^ F(x)= 5+√4 = 7

Q^P^F(x)= $\sqrt{5+4}$= 3

3- Linear Operators

Example4

If $F(X) = X^2 + 2x$, P^= d/dx

$P\hat{}F(x) = \partial/\partial x\ (X^2+2X) = \partial X^2/\partial x + \partial 2X/\partial x$

=2X+2= 2X+2

4- non-Linear Operators

P^(R+S) ≠ P^R + P^S

Example5

If P^= √ , R=5 , S= 10

P^(R+S) = P^(5+10) = $\sqrt{5+10} = \sqrt{15}$

P^ (R+S)= P^R + P^S = $\sqrt{5} + \sqrt{10}$

Example6

If A^= $\partial^2/\partial^2X + 3X\,\partial/\partial X$, F(x)= $4X^3$

A^F(x)= $\partial^2/\partial^2X + 3X\partial/\partial X)4X^3$

$=\partial^2/\partial^2X\ 4X^3 + 3X\partial/\partial x\ 4X^3 = 24X + 3X(12X^2)$

$=36X^3 + 24X$

Example 7

If A^= 5 , F(x) = sin x

A^F(x)= 5 sinx

Example8

If A^= $\partial/\partial x$, F(x)= $\cos(X^2+1)$

A^F(x)= $\partial/\partial x \cos(X^2+1)$

$= -2X \sin(X^2+1)$

Eigen Value Equations

Suppose that effect of operating on some function F(x) with the operator A^ is simply to multiply F(x) by a certain constant K .

We then say that F(x) is an eigen function of A^ with eigen value K.

As part of the the definition .we shall require that the eigen function F(x) is not identically zero.

A^F(x) = K F(x)

Example 1:

e^{2x} is an eigen function of the operator $\partial/\partial x$ with Eigen value 2

$$\partial/\partial x \quad e^{2x} \quad = \quad 2 \quad e^{2x}$$

(operator)(function)=(constant)(same function)

$\partial/\partial x$= operator , e^{2x} = eigen function , 2= eigen value

Example 2:

If the function (cos ax)an eigen function of a) $\partial/\partial x$, b) $\partial^2/\partial^2 x$

Sol/

a)d/dx cos a x = -a sin ax…..non an Eigen function

b)d^2/d^2x cos ax = - a^2 cos ax ……an Eigen function

Example 3:

The function X^3 and the operator $A\hat{} = \partial^2/\partial^2 X$, $B\hat{} = X^2$ find Eigen function ?

Sol/ $A\hat{}B\hat{}X^3 = \partial^2/\partial^2 x\ X^2X^3 = \partial^2/\partial^2 x\ X^5 = 20X^3$

Example 4:

Is wave function $Y(x)= e^{nx2}$ an eigen function for the operator

$(\partial^2/\partial^2 x) -4n^2x^2$?

Sol/

$P\hat{}Y(x)= (\partial^2/\partial^2 x) -4n^2x^2)\ e^{nx2}$

$=\partial^2/\partial x^2\ e^{nx2} -4\ n^2x^2\ e^{nx2}$

$=4\ n^2x^2\ e^{nx2} + 2ne^{nx2} - 4\ n^2x^2\ e^{nx2}$

$= 2ne^{nx2}$ Eigen function

Example 5:

What is Eigen value if wave function $\Psi_{(z)}=Ce^{5z}$,and operator

$P\hat{} = -ih\ \partial/\partial z$

$P\hat{}\ \Psi_{(z)} = -ih\ \partial/\partial z\ Ce^{5z}$

$=-ih5Ce^{5z}$

∴ Eigen value = -5ihC