

Quantum Mechanics

Variable Separation Method



Variable Separation Method

The method of separation of variables combined with the principle of superposition is widely used to solve initial boundary value problems involving Linear partial differential equations .

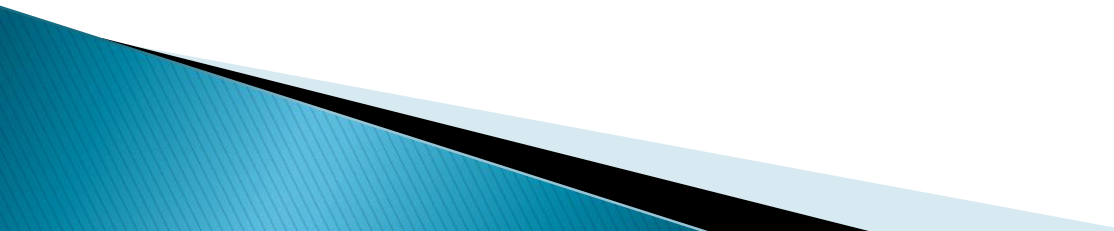
Usually , the dependent variable $U(x, y)$ is expressed in the separable form

$$U(x, y) = X(x) Y(y) \blacktriangleright$$

Where X and Y are functions of x and y . \blacktriangleright

In many cases , the partial differential \blacktriangleright
equation reduces to two ordinary differential
equations for X and Y .

The method is widely used in finding \blacktriangleright
solutions of a large class of initial boundary
value problems .



This method of solution is also as the method ▶
Eigen function expansion .

$$U(x, y) = X(x) Y(y) \quad \blacktriangleright$$

$d U(x, y) / dx - d U(x, y) / dy = 0$ by ▶
substitution where X is function of x and Y is
function of y .

$$dX(x) dY(y) / dx - dX(x) dY(y) / dy = 0 \quad \text{by} \quad \blacktriangleright$$

divided $X(x) Y(y)$

$$1 / X(x) dX(x) / dx - 1 / Y(y) dY(y) / dy = 0 \quad \blacktriangleright$$

$$1 / X(x) dX(x) / dx = C \blacktriangleright$$

$$dX(x) / X(x) = C dx \blacktriangleright$$

$$\ln X(x) = C X + G \blacktriangleright$$

$$X = e^{CX+G} \dots\dots\dots X = Ae^{CX} \blacktriangleright$$

$$1 / Y(y) dY(y) / dy = C \blacktriangleright$$

$$dY / Y(y) = C dy \blacktriangleright$$

$$\ln Y = Cy + R \blacktriangleright$$

$$Y = e^{CY + R} \dots\dots Y = B e^{CY} \blacktriangleright$$

A, B are constant $\dots\dots AB = D$

$$U(x, y) = X(x) Y(y) \blacktriangleright$$

$$U(x, y) = A e^{CX} \cdot B e^{CY} = AB e^{C(X+Y)} \blacktriangleright$$

$$U(x, y) = D e^{C(X+Y)} \blacktriangleright$$

Classical Mechanics:

The classical mechanics describes the behavior of objects in terms of two equations

.

One equation expresses the fact that the total energy is constant in the absence of external forces .

The other equation expresses the response of particles to the forces on them .

Velocity = $X_2 - X_1 / t_2 - t_1 = dx / dt = X \cdot$ ▶

Acceleration = $X \cdot_2 - X \cdot_1 / t_2 - t_1 = d^2X / dt^2 = dX \cdot / dt = X \cdot \cdot$ ▶

$F(x) = m X \cdot \cdot$ ▶

$F =$ force , $m =$ mass , $X \cdot \cdot =$ acceleration ▶

$F(q) = m q \cdot \cdot$, $q = x, y, z$ ▶

Total energy of a particle is the sum of the kinetic energy of the particle , and potential energy (V) the energy arising from the position of the particle in a field of force .

$$E_{(T)} = E_k + E_p$$

$$E_k + E_p = \text{constant}$$

$$T + V = \text{constant}$$

The force (F) is related to the potential energy by

$$F(x) = - \frac{dV}{dX} \blacktriangleright$$

The kinetic energy of a particle of mass (m) travelling with a speed v is :

$$E_k = \frac{1}{2} m v^2 \blacktriangleright$$

$$F(x) = m \frac{d^2x}{dt^2} \blacktriangleright$$

$$F(x) = m \frac{dV}{dt} \blacktriangleright$$

$$m \frac{dV}{dt} = - \frac{dV}{dX} \blacktriangleright$$

$$m \frac{dV}{dt} \frac{dX}{dX} = - \frac{dV}{dX} \blacktriangleright$$

▶ $mX. dX. = - d V$ by integration ▶

$$m \int X. dX. = - \int dV$$

$$m X.^2 / 2 = -V + C \quad \blacktriangleright$$

$$1 / 2 m X.^2 + V = C \quad \blacktriangleright$$

$$T + V = \text{constant} \quad \blacktriangleright$$