Quantum Mechanics

Variable Separation Method

Variable Separation Method

- The method of separation of variables combined with the principle of super position is widely used to solve initial boundary value problems involving Linear partial differential equations .
- Usually , the dependent variable U (x, y) is expressed in the separable form

- $U(x, y) = X(x)\,Y(y)$
- Where X and Y are functions of x and y .
- In many cases , the particle defferential equation reduces to two ordinary differential equations for X and Y .
- The method is widely used in finding solutions of a large class of initial boundary value problems .

- This method of solution is also as the method Eigen function expansion .
- U(x, y) = X(x) Y(y)
- d U(x, y)/ dx – d U(x, y)/dy = 0 by substitution where X is function of y and Y is function of y .
- dX(x) dY(y)/ dx – dX(x) dY(y)/dy = 0 by divided X(x) Y(y)
- 1/X(x) dX(x)/ dx – 1/ Y(y) dY(y)/dy = 0

- $1/X(x)\ dX(x)/\ dx = C$
- $dX(x)/X(x) = C\ dx$
- $\ln X(x) = C X + G$
- $X = e^{CX+G}$ ……..$X = Ae^{CX}$
- $1/\ Y(y)\ dY(y)/dy = C$
- $dY/Y(y) = C\ dy$
- $\ln Y = Cy + R$

$Y = e^{CY + R}$ .... $Y = Be^{CY}$

A, B are constant .... $AB = D$

$U(x, y) = X(x)\, Y(y)$

$U(x, y) = Ae^{CX} . Be^{CY} = AB\, e^{C(X+Y)}$

$U(x, y) = D\, e^{C(X+Y)}$

Classical Mechanics:

- The classical mechanics describes the behavior of objects in terms of two equations .
- One equation expresses the fact that the total energy is constant in the absence of external forces .
- The other equation expresses the response of particles to the forces on them .

- Velocity = $X_2 - X_1 / t_2 - t_1$ = dx/ dt = $\dot{X}$
- Acceleration = $\dot{X}_2 - \dot{X}_1 / t_2 - t_1 = d^2X/dt^2 = d\dot{X}/dt = \ddot{X}$
- F(x)= m $\ddot{X}$
- F= force , m= mass , $\ddot{X}$ = acceleration
- F(q) = m $\ddot{q}$, q= x, y, z

- Total energy of a particle is the sum of the kinetic energy of the particle , and potential energy (V) the energy arising from the position of the particle in a field of force .
- $E_{(T)} = E_k + E_p$
- $E_k + E_p =$ constant
- $T + V =$ constant
- The force (F) is related to the potential energy by

$F(x) = -\frac{dV}{dX}$

The kinetic energy of a particle of mass (m) travelling with a speed v is :

$E_k = 1/2\ m\ v^2$

$F(x) = m\ d^2x/\ dt^2$

$F(x) = m\frac{dX.}{dt}$

$m\frac{dX.}{dt} = -\frac{dV}{dX}$

$m\frac{dX\ dX.}{dt} = -\ dV$

$mX.\ dX. = -\ d\,V$ by integration

$m\int X.\ dX. = -\int dV$

$m\,X^{\cdot 2}/2 = -V + C$

$1/2\ m\,X^{\cdot 2} + V = C$

$T + V = \text{constant}$