Lagrangian and Hamiltonian Function

- The Lagrangian a function of the position or coordinates (qi) and velocities (v) and time of mechanical system .
- L(qi, v, t)
- The momenta are calculated by differentiating the Lagrangian with respect to velocities .
- Pi(qi, v ,t) = dL/ d v

- The Lagrangian is the difference between the potential and the kinetic energy .
- $L = E_k - E_p$
- L= T–V

- Then $E_k = 1/2\ m\ v^2$, $E_p = V$
- $L = 1/2\ m\ v^2 - V$

L= T- V , $L = E_k - E_p$

The equations of motion are given by the Euler – Lagrange equation $\frac{dL}{dv} = p$

- $p = m\,v$, $\frac{d}{dt}\frac{dL}{dv} - \frac{dL}{dqi} = 0$

- lets, convert a Lagrangian system into the equivalent Hamiltonian system .
- Hamiltonian mechanics comes from its application on a one -dimensional system consisting of one particle of mass(m) the Hamiltonian represent the tatal energy of the system ..
- Which is the sum of kinetic and potential energy .

- Given the Lagrangian L(qi, v) .
- Let p= $\frac{dL}{dv}$
- The Hamiltonian is calculated using the usual definition of H as the Legendre transformation of L,
- $H(q_i , p) = \Sigma p v - L (q_i , v)$
- qi=coordant , p= momenta
- $p= m v$, $L= 1/2\, m v^2 - V$
- $H =\Sigma\, m v v- 1/2\, m v^2+ V$
- $H= 1/2\, m v^2 + V$
- $H = T + V$

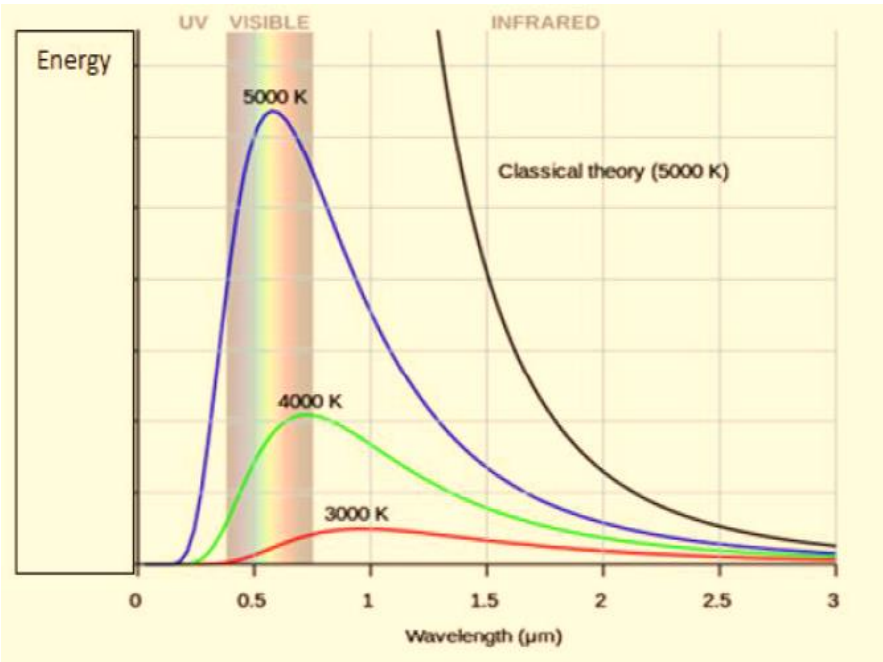
The failure of classical physics

1-Black Body Radiation

- A black body is an object that absorbs all light failing on it .

When physicist used statistical mechanics and the electromagnetic - wave model of light to predict the intensity - versus frequency curve for emitted black body radiation .

- The found a result in complete disagreement with the high - frequency portion of the experimental curves.
- Figure (1) show that the peak in the energy out out shifts to shorter wave lengths the temperature is raised .



- As a result , the short wave length tail of the energy distribution strengthens in the visible region .
- As the temperature decrease the peak of the black body radiation curve motion to lower intensities and lower wave lengths.

A black body at room temperature appears black as most of the energy it radiates is infra red and cannot be perceived by the human eye .

Because the human eye cannot eye perceive light waves at lower frequencies , a black body , viewed in the dark at the lowest just faintly visible temperature .

Color and Temperture

Temperature (°C)	Color
480	Barely red in the dark
600	Dark red
800	Cherry red
950	Orange, barely visible in sunlight
1100	Orange-yellow, visible in bright sunlight
1300	Light yellow, nearly blinding, welding goggles required.
1500	Nearly white, blinding

An analysis of the data led Wilhelm Wien law

$T\lambda_{Max}$= constant

$T\lambda_{Max}$= 0.288 K. cm

- Example 1. The effective temperature of the Sun is 5778 K. What is the value of ƛ_max for the Sun?
- Sol/

ƛ_max = 498.4nm

The physicist Rayleigh studied it the erotically from classical and thought of the electromagnetic field as a collection of oscillators of all possible frequencies .

He regarded the presence of radiation of frequency (v) as signifying that the electromagnetic oscillator of that frequency had been excited .he arrived at the Rayleigh law

$d E = p\, d\lambda$ ......$p = 8\pi KT/ \lambda^4$

Where p is the proportionality ƛand the energy constant between

density in that range of wave lengths , K is the Boltzmann constant K= 1.381x 10−23 J.K−1 .

Rayleigh law is quite successful at long wave lengths (low frequencies) , it fails body at short wave lengths (high frequencies).

Max Planck developed a theory that gave excellent agreement with the observed black body -radiation curves.

- Plank assumed that the atom of the black body could emit light energy only in a mounts given by hv where v is the radiations frequencies , and h is a proportionality constant called Planck's constant the value $h = 6.6x10^{-34}$ J.s give curves that agreed with the experimental black body curves .

Planck's, work the beginning of quantum mechanics .

- Max Planck found that he could account for the experimental observations by proposing that the energy of each electromagnetic oscillator is limited to discrete values and cannot be varied arbitrarily.

The limitation of energies to discrete values is called the quantization of energy .

Planck found that he could account for the observed distribution of energy if he supposed that permitted energies of an electromagnetic oscillator of frequency v are integer multiples of hv .

Planck was able to derive the Planck distribution .

$d E = p\, d\lambda$ $\quad \ldots p = 8\pi hc/ \lambda^5(1/e^{hc/\lambda KT} - 1)$

$E = h\nu$.

number of photon= $\frac{E\ total}{h\nu}$ = $\frac{\rho\ t}{h\nu}$

$$\text{no. of photon } = \frac{\rho\ t\lambda}{hc}$$

Ex1/ calculate the number of photon emitted by a 100 W yellow lamp in 1.0 s take the wave length of yellow light as 560nm ?

Sol/

no. of photon $= \frac{\rho\ t\lambda}{hc} = \frac{560\ x10-9\ m\ x\ 100\ J.s\ x\ 1.0s}{6.626x10-34\ J.s\ x3x10\ 8m.s-1}$

$= 2.8x10^{20}$

Ex2/ calculate the average power out of photo sensitive plate that collects $1.20x10^8$ photons in 5.9 m.s from monochromatic light of wave length 297 nm ?

Sol/ no. of photon $= \frac{\rho\ t\lambda}{hc}$

$1.2x\ 10^{8} = \frac{297\ x10-9\ m\ x\ p\ x\ 5.9x\ 10-3s}{6.626x10-34\ J.s\ x3x10\ 8m.s-1}$

$= 136x10^{-10}\ J.s^{-1} = W$