Fundamentals of Thermodynamics
Lecture 9. Carnot Cycle

9.1. Introduction

A cyclic process is a series of operations by which the state of a substance changes but finally returns to its original state.

If the volume of the working substance changes, *the substance may do external work*, or work may be done on the working substance, during a cyclic process.

Since the initial and final states of the working substance are the same in a cyclic process, and internal energy is a function of state, *the internal energy of the working substance is unchanged in a cyclic process*.

Therefore, the net heat absorbed by the working substance is equal to the external work that it does in the cycle.

If during one cycle of an engine a quantity of heat Q_1 is absorbed and heat Q_2 is rejected, the amount of work done by the engine is $Q_1 - Q_2$ and its *efficiency* η is defined as:

$$\eta = \frac{\text{Work done by the engine}}{\text{Heat absorbed by the working substance}} = \frac{Q_1 - Q_2}{Q_1}$$

Carnot was concerned with the efficiency with which heat engines can do useful mechanical work. He envisaged an ideal heat engine consisting of a working substance contained in a cylinder (Fig. 9.1).

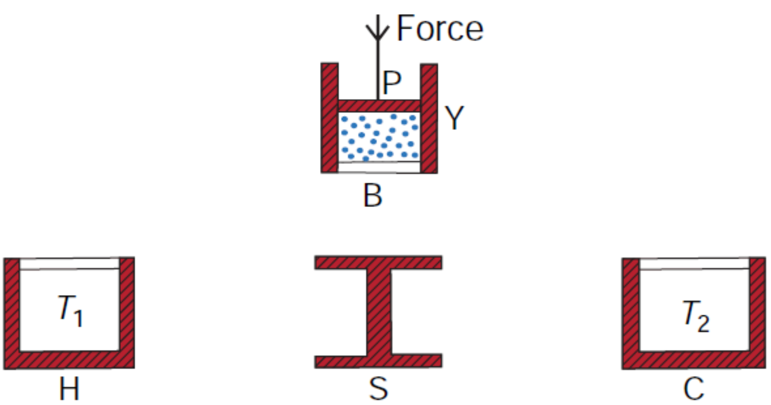


Figure 9.1 The components of Carnot's ideal heat engine

By means of this design, the working substance undergoes transformations which are either *adiabatic* or *isothermal*.

An infinite warm reservoir of heat (H) at constant temperature T_1, and an infinite cold reservoir for heat (C) at constant temperature T_2 (where $T_1 > T_2$) are available. Also, an insulating stand S to facilitate adiabatic changes.

Heat can be supplied from the warm reservoir to the working substance contained in the cylinder, and heat can be extracted from the working substance by the cold reservoir.

As the working substance expands, the piston moves outward and external work is done by the working substance. As the working substance contracts, the piston moves inward and work is done *on* the working substance.

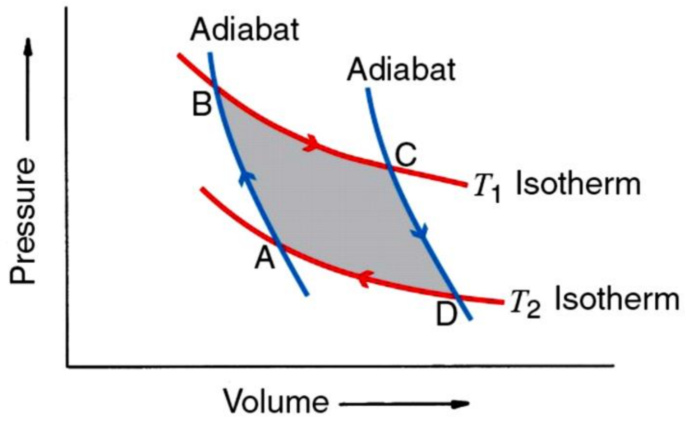


Figure 9.2 Representations of a Carnot cycle on a $p - V$ diagram. The red lines are isotherms and the blue lines adiabats.

Carnot's cycle consists of taking the working substance in the cylinder through the following four operations that together constitute a reversible, cyclic transformation

1. The substance starts at point A with temperature T_2. The working substance is compressed adiabatically to state B. Its temperature rises to T_1.
2. The cylinder is now placed on the warm reservoir H, from which it extracts a quantity of heat Q_1. The working substance expands isothermally at temperature T_1 to point C. During this process the working substance does work.

3. The working substance undergoes an adiabatic expansion to point D and its temperature falls to T_2. Again the working substance does work against the force applied to the piston.
4. Finally, the working substance is compressed isothermally back to its original state A. In this transformation the working substance gives up a quantity of heat Q_2 to the cold reservoir.

The net amount of work done by the working substance during the Carnot cycle is equal to the area contained within the figure ABCD. This can be written

$$W = \oint_C pdV$$

Since the working substance is returned to its original state, the net work done is equal to $Q_1 - Q_2$ and the efficiency of the engine is given by:

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

In this cyclic operation the engine has done work by transferring a certain quantity of heat from a warmer (*H*) to a cooler (*C*) body.

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{|Q_2|}{|Q_1|} = 1 - \frac{T_2}{T_1}$$

One way of stating the Second Law of Thermodynamics is:

"only by transferring heat from a warmer to a colder body heat can be converted into work in a cyclic process."

- Efficiency is 0 if $T_1 = T_2$
- Efficiency is 100 % only if $T_2 = 0$ K but Such reservoirs are not available because Efficiency is always less than 100 %.
- The efficiency increases as T_2 is lowered and as T_1 is raised.
- In most practical cases, T_2 is near room temperature, 300 K.

  So generally T_1 is raised to increase efficiency.

Ref.

[1] Borgnakke, C. and R. E. Sonntag, 2009: Fundamentals of Thermodynamics. Willy Inc., 7th ed. , USA.

[2] Wallace, J. M and P. V. Hobbs, 2006: Atmospheric Science, An Introductory Survey, 2nd ed. , Academic Press, USA.

[3] Visit the website: https://maths.ucd.ie/met/msc/fezzik/Phys-Met/Ch03-P4.pdf

Exercises:

1. The highest theoretical efficiency of a certain engine is 30 %. If this engine uses the atmosphere, which has a temperature of 300 K, as its cold reservoir, what is the temperature of its hot reservoir?

$$\eta = 1 - \frac{T_2}{T_1}$$

$$T_1 = \frac{T_2}{1-\eta} = \frac{300\ \text{K}}{1-0.3} = 429\ \text{K}$$

2. If the temperature of the hot reservoir is increased, the efficiency of the Carnot engine: a. does not change b. decreases c. increases d. will be equal to the efficiency of the practical engine

   Sol. ??

3. A Carnot refrigetor operates between two heat reservoirs whose temperatures are 0 ^oC and 25 ^oC.
   a. What is the coefficient of performance of the refrigerator?
   b. If 200 J of work are done on the working substance per cycle, how much heat per cycle is extracted from the cold reservoir?
   c. How much heat per cycle is discarded to the hot reservoir?

   Sol.

   a.

$$K_R = \frac{T_c}{T_h - T_c} = \frac{273.15\ K}{298.15\ K - 273.15\ K} = 10.9$$

   b.

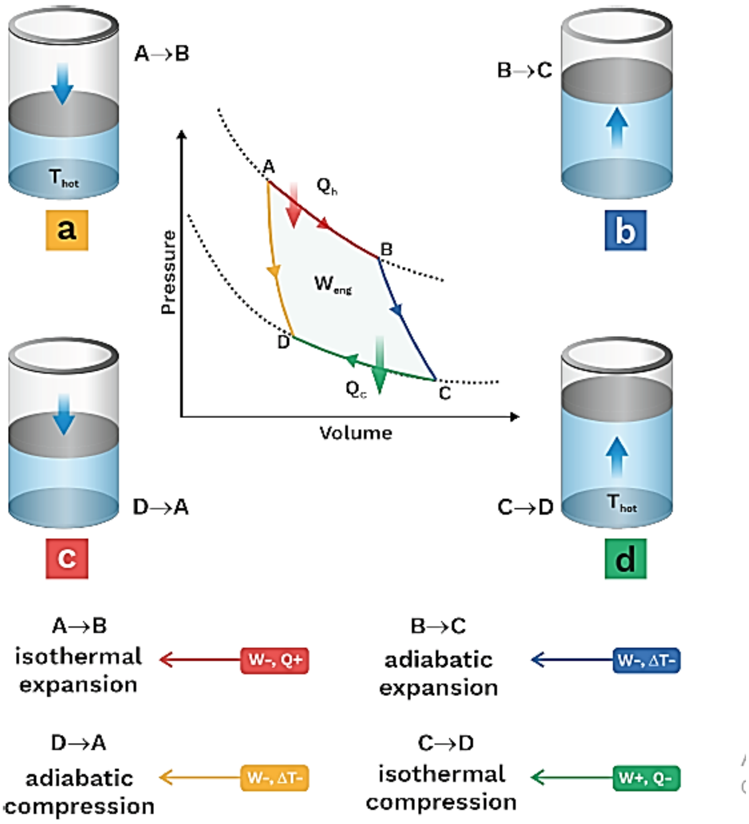
$$Q_c = K_R W = 2.18\ kJ$$

   c.

$$Q_h = Q_c + W = 2.38\ kJ$$

4. Draw Carnot Cycle and show the adiabatic and isothermal expansion and compression process and determine the signs of work and heat.

Sol.



Homework: A heat engine does 9200 J of work per cycle while absorbing 25.0 kcal of heat from a high-temperature reservoir. What is the efficiency of this engine?

Frequently Asked Questions – FAQs

Q1. What is a working fluid in Carnot's cycle?

The working fluid in a Carnot's cycle is an ideal gas.

Q2. What is a Carnot heat engine?

Carnot heat engine is a theoretical engine that operates on a reversible Carnot cycle. It has maximum efficiency that a heat engine can possess.

Q3. Name the processes involved in the Carnot cycle?

It involves four process: isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression.

Q4. What happens in the isothermal expansion process?

In the isothermal expansion process, gas is taken from P_1, V_1, T_1 to P_2, V_2, T_2. Heat Q_1 is absorbed from the reservoir at temperature T_1. The total change in internal energy is zero and the heat absorbed by the gas is equal to the work done.

Q5. Is Carnot's cycle reversible?

The Carnot heat-engine cycle described is a totally reversible cycle.