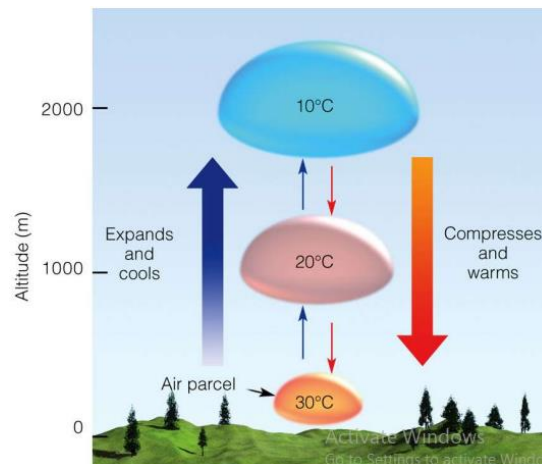# Atmospheric Thermodynamics
Lec. 2 Dry Air Thermodynamics

2.1 Air Parcel

- In atmospheric thermodynamics, the concept of "air parcel" is often used. Air parcel is a hypothetical volume of air that does not mix with its surroundings. It is a good approximation to consider the air parcel as a closed system.

- Parcel can exchange energy with the surrounding by work of expansion or contraction, or by exchanging heat.

- Air parcel moves adiabatically if there is no exchange of heat with surroundings.

- Hence, if the parcel is closed and moves adiabatically, then it is isolated.

- The first principle of thermodynamics states that the *internal energy U* of a closed system, the kinetic and potential energy of its components, is a state variable, depending only on the present state of the system, and not by its past (not a path variable).

- An air parcel can exchange heat with its surroundings through diffusion or thermal conduction or radiative heating or cooling; moreover, evaporation or condensation of water and subsequent removal of the condensate promote an exchange of latent heat.

- It is clear that processes which are not adiabatic ultimately lead the atmospheric behaviors. However, for timescales of motion shorter than one day, and

disregarding cloud processes, it is often a good approximation to treat air motion as adiabatic.

> *State variables* can be calculated by exact differentials. A *path variable* is a variable that depends on the sequence of steps that takes the system from the initial state to the final state. This sequence of steps is called the path. Heat and work are examples of path variables.

2.2 Potential Temperature

For adiabatic processes, the first law of thermodynamics, written in two alternative forms:

$$c_v dT + p dv = dq \qquad (1)$$

$$c_p dT - v dp = dq \qquad (2)$$

holds for $dq = 0$. Integrating (1) and (2) and making use of the ideal gas state equation, we get the Poisson's equations:

$$T v^{\gamma-1} = constant \qquad (3)$$

$$T p^{-k} = constant \qquad (4)$$

where $\gamma = c_p/c_v = 1.4$ and $\kappa = (\gamma - 1)/\gamma = R/c_p \approx 0.286$, using a result of the kinetic theory for diatomic gases. We can use equ. (4) to define a new state variable that is conserved during an adiabatic process, the potential temperature θ, which is the temperature the air parcel would attain if compressed, or expanded, adiabatically to a reference pressure p_0, taken for convention as 1000 hPa.

$$\theta = T(\frac{p_0}{p})^\kappa \qquad (5)$$

θ can be considered conserved along the air motion for one week or more. The distribution of θ in the atmosphere is determined by the pressure and temperature fields.

An adiabatic vertical displacement of an air parcel would change its temperature and pressure in a way to preserve its potential temperature. It is interesting to derive an

expression for the rate of change of temperature with altitude under adiabatic conditions:

using the hydrostatic equation,

$$\frac{\partial p}{\partial z} = -\rho g \qquad (6)$$

and equation of state,

$$p = \rho RT \qquad (7)$$

we can write equ. (2) as:

$$c_p dT + g dz = 0 \qquad (8)$$

and obtain the *dry adiabatic lapse* rate Γ_d:

$$\Gamma_d = -\left(\frac{dT}{dz}\right)_{adiabatic} = \frac{g}{c_p} \qquad (9)$$

If the air parcel thermally interacts with its environment, the adiabatic condition no longer holds and in equs. (1) and (2) $dq \neq 0$. In such case, dividing (2) by T and using $(pV = mRT)$ we obtain:

$$d \ln p - \kappa \, d \ln p = -\frac{dq}{c_p T} \qquad (10)$$

Combining the logarithm of equns. (5) with (10) yields:

$$d \ln \theta = \frac{dq}{c_p T} \qquad (11)$$

That clearly shows how the changes in potential temperature are directly related to the heat exchanged by the system.

2.3 Entropy and Potential Temperature

The second law of the thermodynamics allows for the introduction of another state variable, the *entropy* s, defined in terms of a quantity dq/T which is not in general an exact differential, but is so for a reversible process, that is a process proceeding through states of the system which are always in equilibrium with the environment. Under such cases we may write $ds = (dq/T)_{rev}$. For the generic process, the heat absorbed by

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the system is always lower that what can be absorbed in the reversible case, since a part of heat is lost to the environment. Hence, a statement of the second law of thermodynamics is:

$$ds \geq \frac{dq}{T} \qquad (12)$$

If we introduce (11) in (12), we note how such expression, connecting potential temperature to entropy, would contain only state variables. Hence equality must hold and we get:

$$d \ln \theta = \frac{ds}{c_p} \qquad (13)$$

That directly relates changes in potential temperature with changes in entropy. We stress the fact that in general an adiabatic process does not imply a conservation of entropy. A classical textbook example is the adiabatic free expansion of a gas. However, in atmospheric processes, adiabaticity not only implies the absence of heat exchange through the boundaries of the system, but also absence of heat exchanges between parts of the system itself, that is, no turbulent mixing, which is the principal source of irreversibility. Hence, in the atmosphere, an adiabatic process always conserves entropy.