

## Lecture (17)

### Vertical Coordinate Systems

#### 17.1 Preamble

A model's vertical structure is as important in defining the model's behavior as the horizontal structure and model type.

- To represent the vertical structure of the atmosphere properly requires selection of a suitable vertical coordinate and sufficient vertical resolution.
- Unlike the horizontal structure of models where discrete or continuous (grid point or spectral) configurations can be used, virtually all operational models use discrete vertical structures.

#### 17.2 Sigma Vertical Coordinate

- The equations of motion have their simplest form in pressure coordinates.
- Unfortunately, pressure coordinate systems are not suited to solving the forecast equations because, like height surfaces, they can intersect mountains and consequently 'disappear' over parts of the forecast domain.
- To deal with this problem Phillips (1957) developed a terrain-following coordinate called the sigma ( $\sigma$ ) coordinate.
- The sigma coordinate is used in the NGM, GFS, ECMWF, NOGAPS, and UKMET models and appear in some mesoscale models, such as MM5.
- The sigma coordinate is defined by

$$\sigma = p/p_s \quad (17.1)$$

where  $p$  is the pressure on a forecast level within the model and  $p_s$  is the pressure at the earth's surface, not mean sea level pressure.

- The lowest coordinate surface (usually labeled  $\sigma = 1$ ) follows the actual terrain. Note that the terrain slopes used in sigma models are always smoothed to some degree.
- The other sigma surfaces gradually transition from being nearly parallel to the smoothed terrain at the bottom of the model ( $\sigma = 1$ ) to being nearly horizontal to the constant pressure surface at the top of the model ( $\sigma = 0$ ).
- The sigma vertical coordinate can also be formulated with respect to height ( $z$ ), rather than pressure.

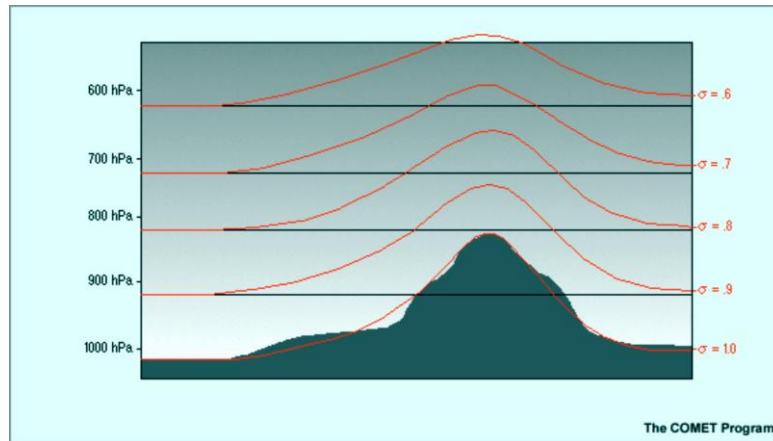


Figure (17.1) Sigma vertical coordinate

### 17.3 Eta Vertical Coordinate

- The eta coordinate ( $\eta$ ) was created in the early 1980s in an effort to reduce the errors incurred in calculating the pressure gradient force using sigma coordinate models.
- The eta coordinate is, in fact, another form of the sigma coordinate, but uses mean sea level pressure instead of surface pressure as a bottom reference level. As such, eta is defined as

$$\eta_s = \frac{P_r(z_s) - P_t}{P_r(z = 0) - P_t} \quad (17.2)$$

where  $P_t$  is pressure at the model top,  $P_r(z = 0)$  is the standard atmosphere MSL pressure (1013 hPa),  $P_r(z_s)$  is the standard atmosphere pressure at the model terrain level  $z_s$ .

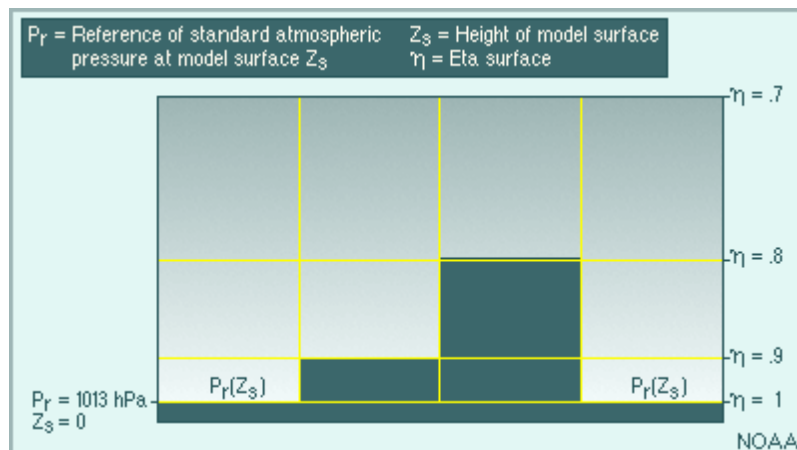


Figure (17.2) Eta vertical coordinate

## 17.4 Calculating Eta Surfaces

- First, the heights at each model level must be defined. In this example, we have defined a model with 10 eta layers (only 3 shown) distributed evenly with respect to pressure from sea level to the top of the atmosphere. Standard atmosphere pressures are then determined at each of these heights.

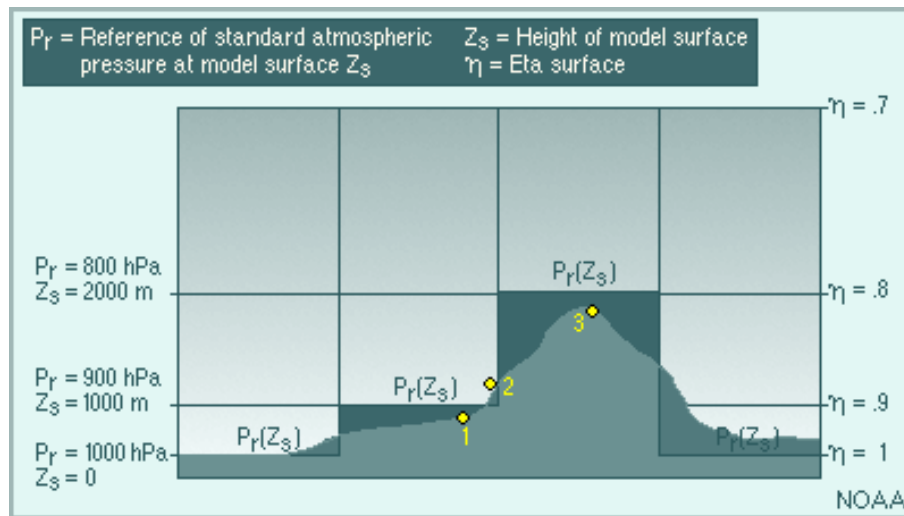


Figure (17.3) Eta vertical coordinate

- At point 1, the actual terrain elevation is 848 m. This is closest to the 1000-m height defined for the first eta level. The standard atmosphere pressure at that height is 900 hPa. What, then, is the eta level closest to this point?

Using the eta equation,

$$\eta = \frac{900 - 0}{1000 - 0} = 0.9$$

If we go to point 2, the actual terrain height is 1126 m and is also closest to the 1000-m height. Therefore, the eta level closest to this point is again 0.9. However, if we go up to point 3 (1832 m), the nearest eta surface in the model is at 2000 m. Here the standard atmospheric pressure is 800 hPa. The nearest eta level is therefore

$$= 1 \times \left( 800 - \frac{0}{1000} - 0 \right) = 0.8$$

This has been a simplified example.

Note that the eta levels are predefined and the model topography is set to the nearest eta surface even if it does not quite match the average or smoothed terrain height in the grid box.

Eta usually is labeled from 0 to 1 from the top of the model domain to mean sea level. The Eta coordinate systems allows the bottom atmospheric layer of the model to be represented within each grid box as a flat "step," rather than sloping like sigma in steep terrain.

## 17.5 Other coordinates

### *1. Isentropic vertical coordinate*

Since flow in the free atmosphere is predominantly isentropic and adiabatic, potential temperature ( $\theta$ ) can be very useful as a vertical coordinate system.

However, non-adiabatic processes dominate in the boundary layer and isentropic surfaces intersect the earth's surface. For these reasons, potential temperature alone is not currently used as a vertical coordinate in any operational numerical model system. However, isentropic coordinates do form an essential part of many hybrid vertical coordinate systems.

### *2. Hybrid vertical coordinates*

Different hybrid combinations are currently in use, e.g.: hybrid isentropic-sigma vertical coordinates, hybrid sigma-pressure coordinate, hybrid isentropic-sigma coordinate.

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## Definitions

*Isentropic* process: is a thermodynamic process, in which the entropy of the fluid or gas remains constant.

*Entropy*: the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of a system.