

Entropy: Is a measure of chaos within a system, and the value of entropy depends on the mass of the system. It is symbolized by (S) and its unit is J/K .

- one of the application of second law of thermodynamics is entropy, while internal energy (U) is deduced from the first law of thermodynamics.

Entropy can also be defined through the second law of thermodynamics as the quantity of heat transferred at transition temperature.

$$dQ = Tds \rightarrow \therefore ds = \frac{dQ}{T}$$

note: the relationship between the first law and second laws of thermodynamics.

$$dQ = dU + dW$$

$$Tds = dU + dW$$

$\therefore$ The relationship between the entropy (S) and internal energy (U):

- In any system that cools, there is a decrease in entropy because internal energy decreases.
- In any system that heat up, there is an increase in entropy because the internal energy increase.

✓ Entropy change has some characteristics :

- 1- It is possible to take the integration for (ds) when the system state changes from initial state (i) to final state (f) -
- 2- For the adiabatic processes $dQ=0$ so, $ds=0$ and $S = \text{constant}$ which is a function for thermodynamic coordinates and it is possible to take the integration for reversible path -
- 3- The change in entropy is perfect differentiation so, it doesn't depend on the path, while only depends on the initial and final states for the system, so, it is suitable to consider the change in entropy as the change in internal energy.
- 4- It is impossible to calculate the absolute value for the entropy (S), while it is possible to calculate the change in entropy (ds).
- 5- The change in entropy doesn't depend on the path, but depends only on initial and final states of the system -

$$\therefore \oint \frac{dQ}{T} = \oint ds = 0$$

$$\therefore \oint \frac{dQ}{T} = \int_i^f \frac{dQ}{T} + \int_f^i \frac{dQ}{T} = 0 \Rightarrow \int_i^f \frac{dQ}{T} = \int_i^f \frac{dQ}{T}$$

$$\Delta S = f(T, V)$$

$$dS = \left(\frac{\partial S}{\partial T} \right)_V dT + \left(\frac{\partial S}{\partial V} \right)_T dV \quad \text{--- } T$$

$$T dS = T \left(\frac{\partial S}{\partial T} \right)_V dT + T \left(\frac{\partial S}{\partial V} \right)_T dV$$

$$T dS = \left(\frac{\partial Q}{\partial T} \right)_V dT + T \left(\frac{\partial P}{\partial T} \right)_V dV$$

$$\therefore \left(\frac{\partial Q}{\partial T} \right)_V = C_V$$

$$\therefore T dS = C_V dT + T \left(\frac{\partial P}{\partial T} \right)_V dV$$

$$\text{For ideal gas} \rightarrow PV = RT \rightarrow P = \frac{RT}{V}$$

$$\left(\frac{\partial P}{\partial T} \right)_V = \frac{R}{V}$$

$$\therefore T dS = C_V dT + T \left(\frac{R}{V} \right) dV \quad \text{--- } T$$

$$\therefore \int dS = C_V \int \frac{dT}{T} + R \int \frac{dV}{V}$$

$$\therefore \int dS = C_V \ln T + R \ln V$$

$$\int dS = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$\Delta s = f(P, V)$$

$$ds = \left[\left(\frac{\partial s}{\partial P} \right)_V dP + \left(\frac{\partial s}{\partial V} \right)_P dV \right] \times T$$

$$T ds = T \left(\frac{\partial s}{\partial P} \right)_V dP + T \left(\frac{\partial s}{\partial V} \right)_P dV$$

$$T ds = \left(\frac{dQ}{dP} \right)_V dP + \left(\frac{dQ}{dV} \right)_P dV$$

$$dQ)_V = C_V dT \quad \text{when } V = \text{constant}$$

$$dQ)_P = C_P dT \quad \text{when } P = \text{constant}$$

$$\therefore T ds = C_V \underbrace{\left(\frac{\partial T}{\partial P} \right)_V}_{\textcircled{1}} dP + C_P \underbrace{\left(\frac{\partial T}{\partial V} \right)_P}_{\textcircled{2}} dV$$

$$\textcircled{1} PV = RT \rightarrow \frac{dT}{dP} = \frac{V}{R}$$

$$\textcircled{2} PV = RT \rightarrow \frac{dT}{dV} = \frac{P}{R}$$

$$\therefore T ds = C_V \left(\frac{V}{R} \right) dP + C_P \left(\frac{P}{R} \right) dV \Big] \div T$$

$$ds = C_V \left(\frac{V}{TR} \right) dP + C_P \left(\frac{P}{TR} \right) dV$$

$$\therefore PV = RT \rightarrow \frac{1}{V} = \frac{P}{TR} \quad \& \quad \frac{1}{P} = \frac{V}{TR}$$

$$\therefore \int ds = C_V \int \frac{dP}{P} + C_P \int \frac{dV}{V}$$

$$\therefore \int ds = C_V \ln P + C_P \ln V$$

$$\Delta S = f(T, P)$$

$$ds = \left(\frac{\partial S}{\partial T} \right)_P dT + \left(\frac{\partial S}{\partial P} \right)_T dP \quad * T$$

$$T ds = T \left(\frac{\partial S}{\partial T} \right)_P dT + T \left(\frac{\partial S}{\partial P} \right)_T dP$$

$$T ds = \left(\frac{dQ}{dT} \right)_P dT - T \left(\frac{\partial V}{\partial T} \right)_P dP$$

$$T ds = C_P dT - T \left(\frac{\partial V}{\partial T} \right)_P dP \quad (\text{ideal})$$

$$T ds = C_P dT - T \left(\frac{R}{P} \right) dP \quad \div T$$

$$\int ds = C_P \int \frac{dT}{T} - R \int \frac{dP}{P}$$

$$\therefore \int ds = C_P \ln T - R \ln P$$