Classical and quantum assemblies :

If the systems in an assembly obey classical mechanics Then alimitation will be imposed on the energies of the systems only if there is adifinite total energy for the assembly. Also each of these classical systems will be completely distinguishable from every other system in the assembly even if the systems all belong to The same species of particle. If, on the other hand the systems in the assembly obey quantum mechanics, there will only be certain discrete energy levels wich are avilable to the systems rather than the continuum of energies wich are avilable to the classical systems. for example in the Case of the simple harmonic oscillator the only values of energy which the oscillator can take are given by $(n+\frac{1}{2})h\nu$ where (n) is an integer, (h) is PlancKs constant and (ν) is the oscillator frequency.

there are two types of quantum mechanical system. If a system has an angular momentum which is half-integral in units of $(h/2\pi)$ or which is equivalent, has an antisymmetric wave function, Then it will obey the Pauli exclusion Principle. Such a system (e.g. an electron or Proton) is known as fermion and will be restricted in its occupation of energy states in that no single state can be occupied by more than one such system. On the other hand, a system wich has an integral value of angular momentum, and hence asymmetric wave function, will not obey the Pauli exclusion Principle. This type of system (e.g. a photon or an alpha Particle) is known as a boson and there is no restriction on the number of such systems wich may occupy a given energy state.

Maxwell - Boltzman Statistics :-

The Purpose of Statistical Physics to enable the macroscopic Properties to be described in terms of the microscopic Properties of the molecules without involving the detailed Calculation of the individual molecules.

Description of the assimbiles - Phase space :-

the state of a system is defined by a six-dimentional space (x, y, z, P_x, P_y, P_z) wich is Termed Phase-space or Γ-space.

$dV = dx\, dy\, dz \Rightarrow$ an element of volume in Encliden space

$d\Gamma = dx\, dy\, dz\, dP_x\, dP_y\, dP_z \Rightarrow$ an element of volume in Phase space

the state of an assembly of (N) systems defined interms of $6N$ coordinates.

$6N$-dimentional Phase space (Γ_{6N} - space)

$$d\Gamma_{6N} = \prod_{i=1}^{i=N} dx_i\, dy_i\, dz_i\, dPx_i\, dPy_i\, dPz_i$$

$$= \prod_{i=1}^{N} (d\Gamma)_i$$

where $(d\Gamma)_i$ is the Volume element of the six-dimentional Phase space for (i)th system.

Distribution over energies:

The distribution of the systems over the various energies wich are avilable

: $\epsilon_1, \epsilon_2, \epsilon_3, --- \epsilon_i --- \epsilon_N$

$$\sum_{i=1}^{i=N} \epsilon_i = E_{total}$$

Consider the energies of the system can be divided into sheets (S), the effective energy of the system in the sheet is (ϵ_s), the number of energy states avilable to the systems in the sheet (S) is (g_s) is called the weight of the sheet, or the degeneracy of the sheet.

the distribution of the systems over the various energies is given by specifying the occupation number (n_s) of systems with energy E_s.

The total of the occupation number.

$$\sum_{s=1}^{r} n_s = N$$

The total energy of the assembly is

$$\sum_{s=1}^{r} n_s E_s = E_{Total}$$

* The over all configuration of the assembly for wich only the numbers of systems in an energy sheet are specified is sometimes refered to as amacrostate.

* The particulare arrangement of the assembly in wich the actual system occupying agiven energy state is specified, are

known as microstate.

Example :

Assume there are four systems, labelled a, b, c and d are distributed over two energy sheets of wight $g=3$ and $g=4$ respectively (distribution over two sheets with two systems in sheet 1 and two system in sheet 2).

Sheet 1 ; $g=3$, $n_s=2$

a	b	

b	a	

a		b

a	d	

Sheet 2 ; $g=4$, $n_s=2$

c	d		

c		a	

a		b	

c			a

Calculate the total number of arrangment (the weight of the configuration) or the total thermodynamics probability W.

Solution:

$$W = N! \prod_s \left(\frac{g_s^{n_s}}{n_s!}\right)$$

$$\therefore W = 4! \prod_s \left[\frac{3^2}{2\times 1} \times \frac{4^2}{2\times 1}\right] = 24 \times 36 = 864$$

calculation show that, in the classical case where the systems are completely distinguishab there are (864) possible arrangements correspon-ding to given configuration.

If the energies of the systems are spread over a total of (r) energy sheets then the distribution may be written in terms of the occupation numbers as follows:

sheet number	1	2	3	---	S	---	r
sheet energy	ϵ_1	ϵ_2	ϵ_3	---	ϵ_s	---	ϵ_r
weight of sheet	g_1	g_2	g_3	---	g_s	---	g_r
occupation number	n_1	n_2	n_3	---	n_s	---	n_r

where the total of the occupation numbers $\sum_{s=1}^{r} n_s$ is equal to the total number of systems, N.

the energy of the systems in the sheet (s) is $(n_s \epsilon_s)$ and the total energy of the assembly is $\left(\sum_{s=1}^{r} n_s \epsilon_s\right)$.

Weight of configuration (W) :

The number of distinct arrangement of the system which all correspond to the particular configuration, (The probability that an assembly is in the given configuration will be proportional to the weight of the configuration).

$N-n_1-n_2$ ——— n_3

$N-n_1$ ——— n_2

N ——— n_1

$${}_{N}C_{n_1} = \frac{N!}{n_1!(N-n_1)!}$$ ⇒ For the first sheet

$${}_{N-n_1}C_{n_2} = \frac{(N-n_1)!}{n_2!(N-n_1-n_2)}$$ ⇒ For the 2nd sheet

* The total number of ways of chossing the configuration (for 3 sheets).

$$W = \left[\frac{N!}{n_1!\, n_2!\, n_3!} \right]$$

* in the energy sheet (S) there are (g_s) energy stats, n_s systems in this sheet may be placed in (g_s) ways of arranging the (n_s) system within the sheet (g_s), the weight of configuration

$$W = \frac{N!}{n_1!\, n_2!\, n_3!} \left(g_1^{n_1}, g_2^{n_2}, \ldots g_s^{n_s} \right)$$

$$W = N! \prod \left[\frac{g_s^{n_s}}{n_s!} \right]$$

Ex: Consider two cells i and j in the phase space and four phase point (a,b,c,d), find the weight configuration.

N_i	4	3	2	1	0
N_j	0	1	2	3	4

$$W = \frac{N!}{\prod N_i!} = \frac{N!}{N_1!\,N_2!\,N_3!}$$

① $\left.\begin{matrix} N_i = 4 \\ N_j = 0 \end{matrix}\right\} \Rightarrow W_1 = \frac{N!}{N_i!\,N_j!} = \frac{4!}{4!\,0!} = 1$

Ni	abcd
Ng	0

② $\left.\begin{matrix} N_i = 3 \\ N_j = 1 \end{matrix}\right\} \Rightarrow W_2 = \frac{4!}{3!\,1!} = 4$

Ni	abc	bcd	cda	dab
Nj	d	a	b	c

③ $\left.\begin{matrix} N_i = 2 \\ N_j = 2 \end{matrix}\right\} \Rightarrow W_3 = \frac{4!}{2!\,2!} = \boxed{6}$ most probable

Ni	ab	bc	cd	da	ac	bd
Nj	cd	ad	ab	bc	bd	ac

④ $\left.\begin{matrix} N_i = 1 \\ N_j = 3 \end{matrix}\right\} \Rightarrow W_4 = \frac{4!}{1!\,3!} = 4$

Ni	a	b	c	d
Nj	bcd	acd	abd	abc

⑤ $\left.\begin{matrix} N_i = 0 \\ N_j = 4 \end{matrix}\right\} \Rightarrow W_5 = \frac{4!}{0!\,4!} = 1$

Ni	0
Nj	abcd

$\therefore$ The total number of possible macrostates = 5

and the total number of possible microstates is :

$$\sum_{K=1}^{5} W_K = W_1 + W_2 + W_3 + W_4 + W_5$$

$$= 1 + 4 + 6 + 4 + 1 = 16$$