The Fermi-Dirac Distribution :-

The systems have half-integral angular momentum and obey the pauli exclusion principle, the individual states can be occupied by asingular system.

The given configuration is then specified by the values of (n_s), the occupation numbers of the sheets for the various values of (s). because of the fixed number of systems and the fixed total energy the conditions.

$$\sum_s n_s = N \quad , \quad \sum_s n_s \epsilon_s = E$$

the total number of arrangements corr-esponding to the given configuration will be again :

$$W = \prod_s W_s$$

where (W_s) is the ways of arranging the (n_s) systems in sheet (s) among the (g_s) states.

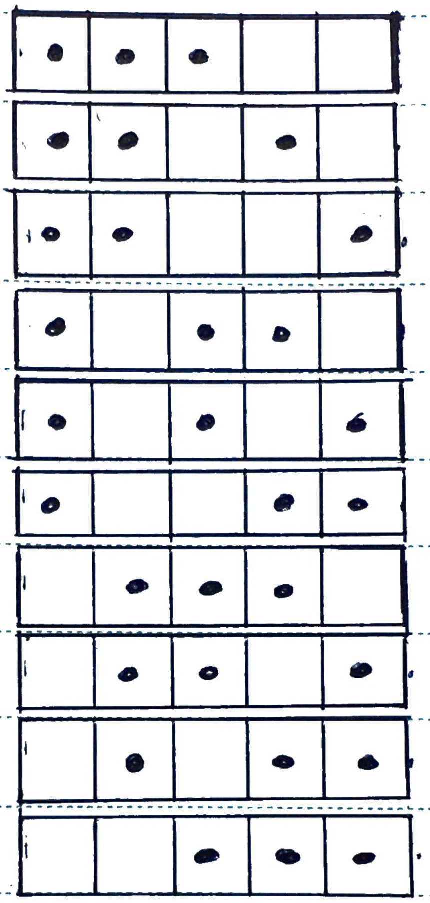
Because, by the exclusion principle, the occupation numbers of a single state can be only (0) or (1) then (n_s) of the g_s states in sheet (s) will be occupied by a system and the other ($g_s - n_s$) of the g_s states will be empty, then

$$W_s = \frac{g_s!}{n_s!\,(g_s - n_s)!} \quad \text{---- (1)}$$

$$\therefore \quad W = \prod_s \frac{g_s!}{n_s!\,(g_s - n_s)!} \quad \text{---- (2)}$$

To illustrate this form of arrangement the figure below shows the number of ways of arranging three filled states among a total of five states. the result is in agreement with equation (1) so:

$$W_s = \frac{5!}{3!\,2!} = 10$$ ways of arranging five states of which three are occupied by dots (systems) and two are empty.



The sheets are chosen to give values of (g_s) and (n_s) sufficiently large for the use of stirling's approximation to be justified, equation ② will give

$$\log W = \sum_s \log \frac{g_s!}{n_s!(g_s-n_s)!}$$

$$= \log g_s! - \log n_s! - \log (g_s - n_s)!$$

$$= g_s \log g_s - \cancel{g_s} - n_s \log n_s + \cancel{n_s} - (g_s - n_s) \log (g_s - n_s) + \cancel{g_s - n_s}$$

$$= g_s \log g_s - n_s \log n_s - (g_s - n_s) \log (g_s - n_s)$$

$$\therefore \frac{\partial \log W}{\partial n_s} = -\cancel{n_s} * \frac{1}{\cancel{n_s}} - \log n_s * 1 + \cancel{(g_s - n_s)} * \frac{1}{\cancel{(g_s - n_s)}} - \log (g_s - n_s) * -1$$

$$\therefore \frac{\partial \log W}{\partial n_s} = \log \left(\frac{g_s - n_s}{n_s} \right) \quad \text{—— (3)}$$

The condition for the most probable configuration is again given by:

$$\sum_s \left\{ \frac{\partial \log W}{\partial n_s} + \alpha + \beta \epsilon_s \right\} dn_s = 0 \quad \text{—— (4)}$$

$$\therefore \frac{\partial \log W}{\partial n_s} + \alpha + \beta \epsilon_s = 0 \quad \text{—— (5)}$$

Substituting (3) in (5)

$$\therefore \log \frac{g_s - n_s}{n_s} + \alpha + \beta \epsilon_s = 0$$

$$\therefore \frac{g_s}{n_s} = e^{-(\alpha+\beta\epsilon_s)} + 1$$

$$\therefore \boxed{n_s = \frac{g_s}{e^{-(\alpha+\beta\epsilon_s)}+1}}$$

The value of (n_s) in the most probable configuration, the Fermi-Dirac distribution.

For an assymbly of (Fremions)

$$f(\epsilon) = \frac{1}{e^{-(\alpha+\beta\epsilon_s)}+1} \quad \text{(fermi function)}$$

$$\therefore \beta = \frac{-1}{KT} \quad , \quad \alpha = \epsilon_F / KT$$

the Fermi function can be more conveniently and written in the form:

$$\boxed{f(\epsilon) = \frac{1}{e^{(\epsilon-\epsilon_F)/KT}+1}}$$

Example (1) :

Calculate the number of possible ways to configurate (8) undistinguishable balls in (10) distinguishable boxes, So that the box doesn't contain more than one ball.

Sol.:

$$\because W_{F.D} = \frac{g_s!}{n_s!\,(g_s-n_s)!}$$

$$\therefore W_{F.D} = \frac{10!}{8!\,(10-8)!} = \frac{10!}{2!\,8!} = 45$$

Example (2):

Consider a system of two identical non-interacting particles, each having three states of energy E, Zero, -E, describe all the state of the system according to M.B, B.E and F.D statistics.

Sol.

① For Maxwell-Boltzman distribution

$$W_{M.B} = N! \prod \left[\frac{g_s^{n_s}}{n_s!} \right] = 2! \frac{3^2}{2!} = 9$$

② For Bose-Einstein distribution

$$W_{B.E} = \frac{[(g_s - 1) + n_s]!}{(g_s - 1)! \, n_s!} = \frac{[(3-1)+2]!}{(3-1)! \; 2!} = \frac{4!}{2! \, 2!} = 6$$

③ For Fermi-Drac distribution

$$W_{F.D} = \frac{g_s!}{n_s! \, (g_s - n_s)!} = \frac{3!}{2! \, (3-2)!} = 3$$

-E	0	+E
AB		
	AB	
		AB
A	B	
A		B
	A	B
B	A	
B		A
	B	A

(M.B)

-E	0	+E
••		
	••	
		••
•	•	
•		•
	•	•

(B.E)

-E	0	+E
•	•	
•		•
	•	•

(F.D)