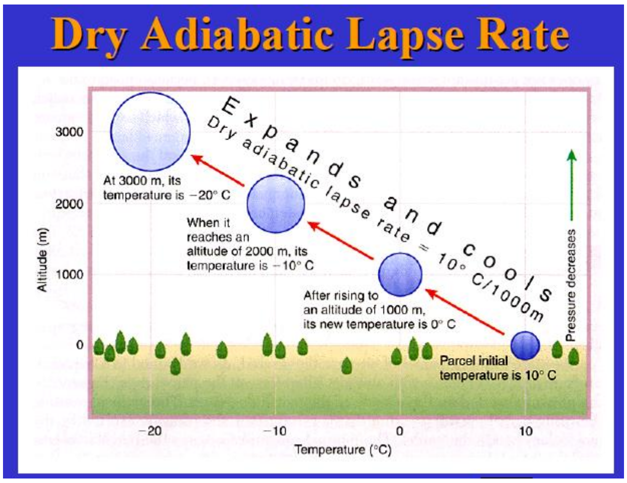
Atmospheric Thermodynamics

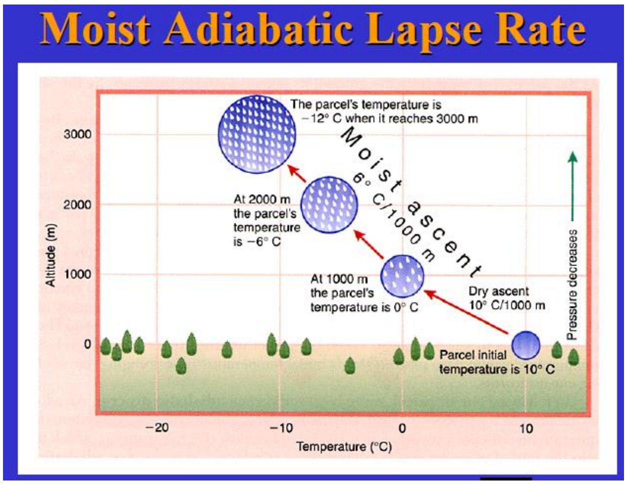
Lecture 6: Atmospheric Stability

6.1 Preface

A. Dry Adiabatic Lapse rate → 10 ^{o}C/1000 m

B. Moist Adiabatic Lapse Rate → 6 ^{o}C/1000 m





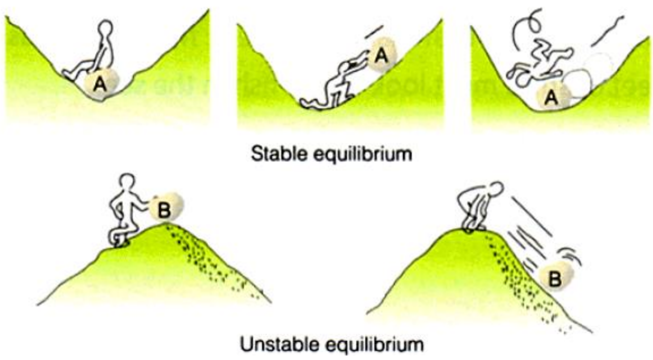
Notes:

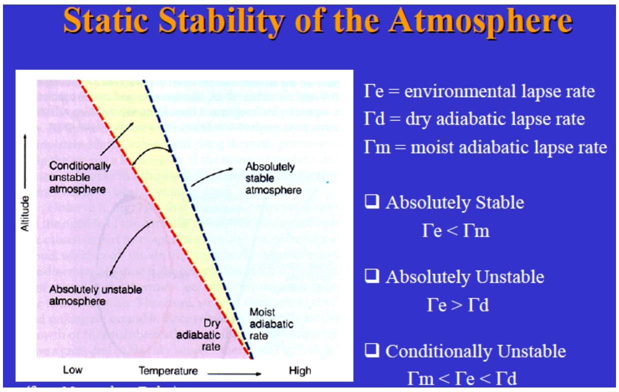
1. In the situation of moist air, the parcel goes up at first with dry adiabatic lapse rate. At the level of condensation, the parcel starts moving up with moist adiabatic lapse rate.
2. Moist adiabatic lapse rate is NOT a constant. It depends on the temperature of saturated air parcel.
3. When warm, saturated air cools, it causes more condensation (and more latent heat release) than for cold, saturated air.

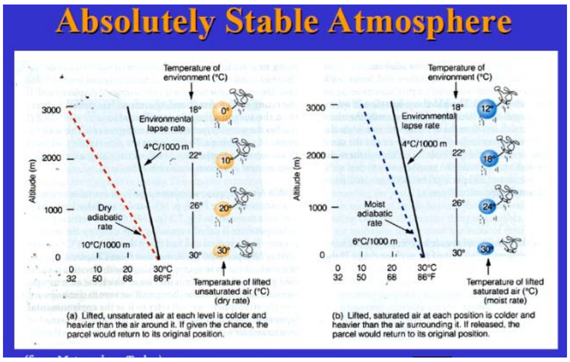
C. Concept of Stability

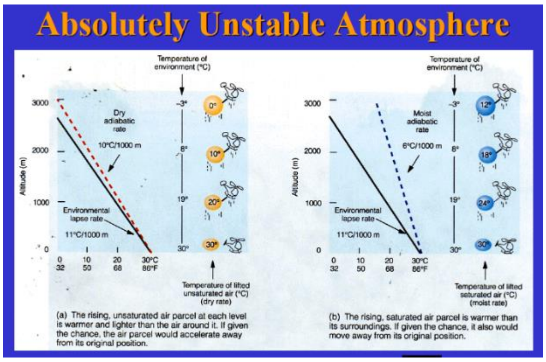
In the case of stable equilibrium, there is a restoring force returns the body (or parcel) to its original place. In the unstable equilibrium, the body (or parcel) does not return to its original place.

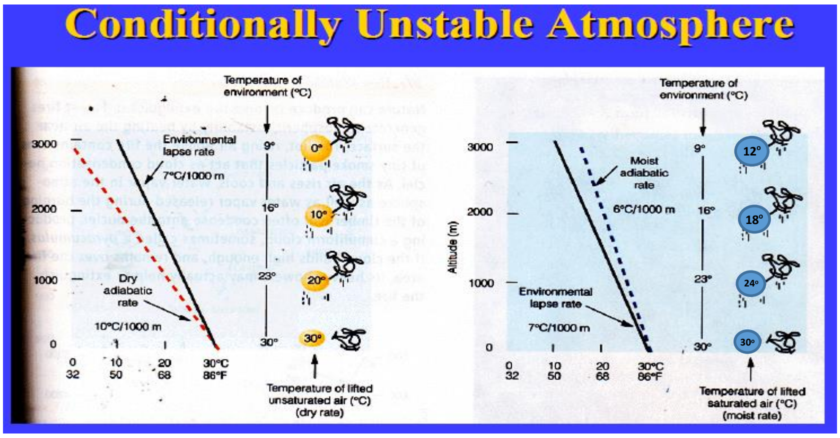
To determine the static stability, we need to compare the lapse rate of the atmosphere (environmental lapse rate) and the dry (moist) adiabatic lapse rate of a dry (moist) air parcel.



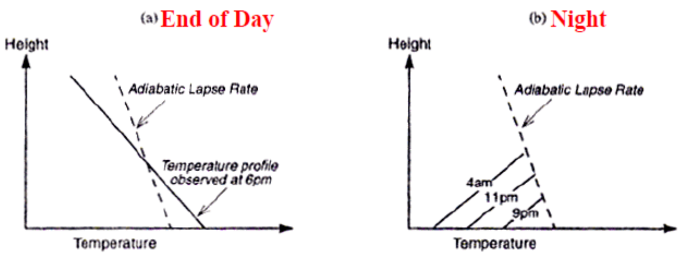






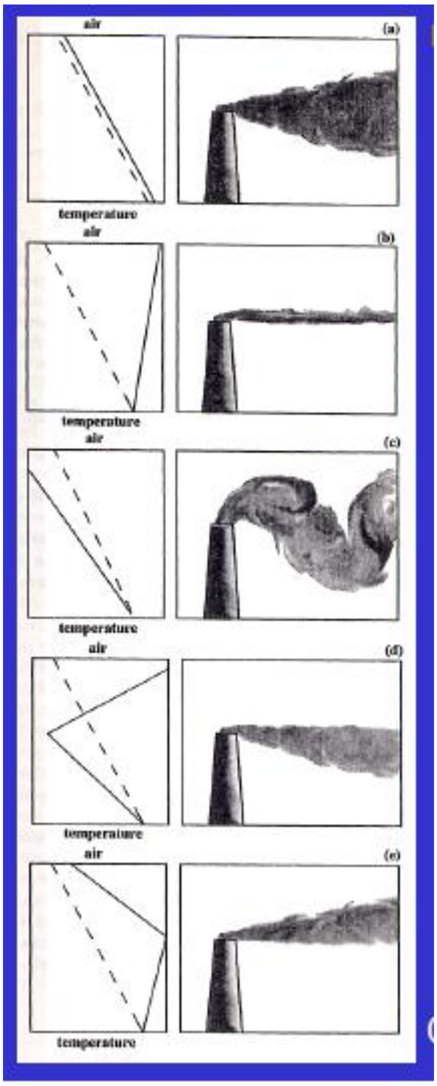


Day/Night Changes of Air Temperature



- ❖ At the end of a sunny day, warm air near the surface, cold air aloft.
- ❖ In the early morning, cold air near the surface, warm air aloft.
- ❖ The later condition is called "inversion", which inhibits convection and can cause severe pollution in the morning.

Stability and Air Pollution



Neutral Atmosphere (Coning)

Stable Atmosphere (Fanning)

Unstable Atmosphere (Looping)

Stable Aloft; Unstable Below (Fumigation)

Unstable Aloft; Stable Below (Lofting)

6.2 Dry Adiabatic Lapse Rate

Consider a parcel having temperature, pressure, and specific volume of $\acute{T}$, $\acute{p}$, and $\acute{\alpha}$. Variables with a prime are properties of the air parcel. Variables without a prime refer to the surrounding, environmental air.

The first law of thermodynamics for the air parcel is

$$c_p d\acute{T} = d\acute{q} + \acute{\alpha}\, d\acute{p}$$

For an adiabatic process this becomes

$$c_p d\acute{T} = \acute{\alpha}\, d\acute{p}$$

We would like to know how the temperature of an air parcel would change if we lift it. What we would like to know is $d\acute{T}/dz$. Dividing the first law by dz, we get for the air parcel

$$c_p \frac{\partial \acute{T}}{\partial z} = \acute{\alpha} \frac{\partial \acute{p}}{\partial z}$$

As the air parcel rises we assume that its pressure, $\acute{p}$, immediately adjusts to that of its environment, p. This, along with the hydrostatic equation, gives

$$\frac{\partial \acute{p}}{\partial z} = \frac{\partial p}{\partial z} = -\frac{g}{\alpha}$$

Therefore, we can write

$$\frac{\partial \acute{T}}{\partial z} = -\frac{\acute{\alpha}}{\alpha} \frac{g}{c_p}$$

The density of the air parcel will be close to that of the density of the air, so $\acute{\alpha} \approx \alpha$. Therefore, we have

$$\frac{\partial \acute{T}}{\partial z} = -\frac{g}{c_p}$$

This formula says that if you lift an air parcel adiabatically, its temperature will decrease, which makes physical sense because the parcel will be expanding.

The dry adiabatic lapse rate is defined as

$$\Gamma_d \equiv -\left[\frac{\partial \acute{T}}{\partial z}\right]_{adiabatic} = \frac{g}{c_p} \qquad \textit{Dry adiabatic Lapse Rate}$$

Recall that lapse rate is defined with a negative sign, so that a positive lapse rate means temperature decreases with height.

For dry air $c_p = 1004\, J\, kg^{-1} K^{-1}$, so that $\Gamma_d = 9.8\, {}^oC/km$ ($\sim 10\, {}^oC/km$).

6.3 Buoyancy

The vertical momentum equation for an air parcel is

$$\frac{\partial \acute{w}}{\partial z} = -\frac{1}{\acute{\rho}} \frac{\partial p}{\partial z} - g$$

From the hydrostatic equation we know that

$$\frac{\partial p}{\partial z} = -\rho g$$

so the momentum equation becomes

$$\frac{\partial \acute{w}}{\partial z} = -\frac{1}{\acute{\rho}} (-\rho g) - g = \frac{(\rho - \acute{\rho})}{\acute{\rho}} g$$

If the parcel is denser than the environment, the acceleration will be downward. If it is lighter than the environment, the acceleration will be upward.

Substituting for density from the ideal gas law, and assuming the pressure of the air parcel is the same as the pressure of the environment ($p = \acute{p}$), we can write the acceleration in terms of temperature

$$\frac{\partial \acute{w}}{\partial z} = a_z = \frac{(\acute{T} - T)}{\acute{T}} g$$

This shows us that warm air rises and cold air sinks.

6.4 Stability in a Dry Atmosphere

Stability refers to whether an air parcel, if moved vertically, will continue to accelerate in the direction that it was pushed (unstable), or return in the direction from which it came (stable).

Imagine a parcel that is in equilibrium with the environment, so that $\acute{T} = T = T_0$. There will be no acceleration of the air parcel.

If the air parcel is displaced, $\acute{T}$ will change according to the adiabatic lapse rate so that $\acute{T}(z) = T_0 - \Gamma_d z$

- At altitude z, the environmental temperature is $T(z) = T_0 - \gamma z$
- The acceleration at altitude z is

$$a_z = \frac{(\gamma - \Gamma_d)}{(T_0 - \gamma z)} g z$$

- If the acceleration is positive (upward), the parcel will continue to accelerate away from its original position, and the air is unstable.
- If the acceleration is negative (downward), the parcel will begin to move downward back toward its original position, and the air is stable.
- If the acceleration is zero, the parcel will remain at its new location, and the air is neutral.

To assess the stability, the environmental lapse rate (γ) must be compared with the dry adiabatic lapse rate. This leads to the following stability criteria:

$\Gamma_d < \gamma$ unstable

$\Gamma_d = \gamma$ neutral

$\Gamma_d > \gamma$ stable

6.5 Potential Temperature

Potential temperature (denoted as θ) is defined as the temperature an air parcel would have if it were moved dry-adiabatically to a reference pressure $p_0 = 1000\ mb$. From Poisson's relation for T and p we get that

$$\theta = T\left[\frac{p_0}{p}\right]^{R_d/c_p}$$

- If the air parcel undergoes an adiabatic process its potential temperature is conserved.
- The potential temperature of the environment can also be used to assess the stability. It turns out that the vertical acceleration of an air parcel can be given in terms of potential temperature,

$$a_z = \frac{(\acute{\theta} - \theta)}{\theta} g$$

To see this, imagine that the air parcel starts out at the same temperature (and therefore potential temperature is as its environment $\theta = \acute{\theta} = \theta_0$). If the parcel is lifted a small distance z, then

$$\theta(z) = \theta_0 + \frac{\partial \theta}{\partial z} z$$

while $\acute{\theta}(z)$ continues to equal θ_0, since potential temperature is conserved in adiabatic motion. Therefore,

$$a_z = -\frac{g}{\theta}\frac{\partial \theta}{\partial z} z$$

The potential temperature profile alone can be used to assess the stability of an unsaturated air parcel.

$$\frac{\partial \theta}{\partial z} > 0 \qquad stable$$

$$\frac{\partial \theta}{\partial z} = 0 \qquad neutral$$

$$\frac{\partial \theta}{\partial z} < 0 \qquad unstable$$

6.6 Brunt-Vaisala Frequency

We have previously shown that the vertical acceleration of an air parcel is

$$a_z = -\frac{g}{\theta}\frac{\partial\theta}{\partial z}z$$

Since we can write $a_z = \frac{\partial^2 z}{\partial t^2}$, we can write

$$\frac{\partial^2 z}{\partial t^2} = -\frac{g}{\theta}\frac{\partial\theta}{\partial z}z$$

or

$$\frac{\partial^2 z}{\partial t^2} + (\frac{g}{\theta}\frac{\partial\theta}{\partial z})z = 0$$

This is a 2nd order homogeneous ordinary differential equation of the form

$$\frac{\partial^2 z}{\partial t^2} + N^2\, z = 0$$

Where

$$N^2 = \frac{g}{\theta}\,\frac{\partial\theta}{\partial z}$$

If N^2 is positive the solution is

$$z(t) = Ae^{iNt} + Be^{-iNt}$$

and the parcel oscillates around its initial altitude at frequency *N*. In this case the atmosphere is stable.

If N^2 is negative the solution is

$$z(t) = Ae^{Nt} + Be^{-Nt}$$

and the parcel accelerates away from its initial altitude. In this case the atmosphere is unstable.

- *N* is known as the Brunt-Vaisala frequency.
- The Brunt-Vaisala frequency is an *angular* frequency, meaning it is actually expressed in radians per second.
- The natural frequency of the parcel's oscillation is $N/2\pi$.

6.7 Dry Static Energy

From the first law of thermodynamics for an adiabatic process we have

$$c_p dT - \alpha\, dp = 0$$

Substituting dp from the hydrostatic equation gives

$$c_p dT + g\, dz = 0$$

which can be written as

$$dE = d(\, c_p T + g\, z) = 0$$

- The quantity $c_p T + g\, z$ is conserved under adiabatic processes in a hydrostatic atmosphere. It is called the *dry static energy*, and is the sum of the specific enthalpy plus potential energy per unit mass.
  (remember that h=u+pv and pot.energy=mgh)
- Dry static energy is an intensive quantity.
- Dry static energy can be related to potential temperature in the following manner: Start with

$$T = \theta \left[\frac{p}{p_0}\right]^{R_d/c_p}$$

and take the differential to get

$$c_p dT = c_p \frac{T}{\theta} d\theta + R_d T \frac{dp}{p} \qquad (1)$$

From the hydrostatic equation and the ideal gas law we can write

$$g dz = -\alpha dp = -R_d T \frac{dp}{p} \qquad (2)$$

Adding Equations (1) and (2) we get

$$c_p dT + g dz = c_p \frac{T}{\theta} d\theta$$

or

$$dE = c_p\, T\, d \ln \theta$$

A commonly used approximation relating dry static energy to potential temperature is

$$E \cong c_p\, \theta$$

This is very accurate and can be used for most, but not all, applications.

EXERCISES

1. A dry air parcel has a temperature of 20 °C. The environmental lapse rate is 5 °C/km. The air parcel is forced to rise over a mountain that is 3 km high.

a. What is the temperature of the air parcel at the top of the mountain?

b. What is the temperature of the environment at the top of the mountain?

c. What is the buoyant acceleration of the air parcel at the top of the mountain?

d. Is the atmosphere stable or unstable?

2. For the following data, find the potential temperature at the two altitudes. Is the atmosphere stable or unstable?

Altitude (m)	Pressure (mb)	Temp (°C)	θ (K)
1400	850	7	
5700	500	-15	

3. Show that the buoyant acceleration of an air parcel can be written as

$$a_z = \frac{(\acute{\theta} - \theta)}{\theta} g$$

4. Start with the ideal gas law and differentiate it with respect to *z*. Show that if the lapse rate is greater than g/R_d then density will increase with height. This lapse rate is known as the *autoconvective* lapse rate.