1. Natural Radioactivity

1. The radioactivity is defined as a phenomenon of spontaneous decay of a nucleus accompanied by the emission of alpha-particles, beta-particles or gamma rays.
2. We cannot predict which nuclei in a sample will decay because all nuclei have the same chance (probability) for decay.

Q1: Derive the law of radioactive decay (or law of exponential decay or law of disintegration).

Answer: If a sample contains N radioactive nuclei, the rate of spontaneous decay of radioactive nuclei $\frac{dN}{dt}$ is proportional to N, i.e., $\frac{dN}{dt} \propto N \rightarrow \frac{dN}{dt} = -\lambda N \rightarrow \therefore \frac{dN}{N} = -\lambda dt$ ....(1)

where the proportionality constant λ is called the decay constant (or disintegration constant). The constant λ (in unit of s^{-1}) has a specific value for every radioactive nucleus. The minus sign in Eq. (1) indicates that as t increases N decreases. Integrating both side of Eq. (1), we obtain:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt \rightarrow \int_{N_0}^{N} \ln N = -\lambda \int_{t_0}^{t} dt \rightarrow \ln N - \ln N_0 = -\lambda (t - t_0) \text{ ....(2)}$$

Here, N_0 is the number of radioactive nuclei in the sample at some arbitrary initial time t_0. Setting $t_0 = 0$ and rearranging Eq. (2), we get: $\ln \frac{N}{N_0} = -\lambda t$ .... (3)

Taking the exponential of both sides of Eq. (3), we get: $\frac{N}{N_0} = e^{-\lambda t} \rightarrow N = N_0 e^{-\lambda t}$ .... (4)

Note that: N_0 is the number of radioactive nuclei in the sample at $t = 0$ and N is the number remaining at any subsequent time t.

Q2: Derive the decay rate R ($R = -\frac{dN}{dt}$) of a sample.

Answer: Differentiating Eq. (4), we find:

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} \rightarrow -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} \rightarrow \therefore R = R_0 e^{-\lambda t} \text{ ....(5)}$$

Note that: Eq. (5) is an alternative form of the law of radioactive decay [Eq. (4)].

In Eq. (5), $R_0 = \lambda N_0$ is the decay rate at time $t = 0$, and R is the rate at any subsequent time t, where

$$R = \lambda N \text{ ....(6)}$$

In Eq. (6), R and the number of radioactive nuclei N that have not yet undergone decay must be evaluated at the same instant.

Q3: Define the activity of the sample.

Answer: It is defined as the decay rate R of a sample (or defined as the number of disintegration per unit time). Its SI unit is in Becquerel (Bq), named for Henri Becquerel, the discoverer of radioactivity.

Note that: **(a)** 1 Bq = 1disintegration per second (s) = 1 dis/s.
(b) An older unit, called Curie (Ci), is still in common use:
1 Ci = 3.7×10^{10} Bq
= 3.7×10^{10} dis/s.

Q4: Define the half-life $t_{1/2}$ of a radionuclide.

Answer: It is defined as the time required at which both N and R are reduced to one-half of their initial values (see Fig. 1).

Q5: Define the mean life τ of a radionuclide.

Answer: It is defined as the time required at which both N and R are reduced to e^{-1} of their initial values.

Q6: How can we relate $t_{1/2}$ to λ? Derive a formula for this relationship.

Answer: We put $R=\frac{1}{2}R_0$ in Eq. (5) and substitute $t_{1/2}$ for t. Solving for $t_{1/2}$ we get:

$$\frac{1}{2}R_0 = R_0 e^{-\lambda t_{1/2}} \quad\rightarrow\quad \frac{1}{2}=e^{-\lambda t_{1/2}} \quad\rightarrow\quad 2=e^{\lambda t_{1/2}}$$

Taking the natural logarithm of both sides and solving for $t_{1/2}$ we find:

$$\ln 2=\lambda t_{1/2} \quad\rightarrow\quad t_{1/2}=\frac{\ln 2}{\lambda} \quad\rightarrow\quad \therefore\ t_{1/2}=\frac{0.693}{\lambda}$$

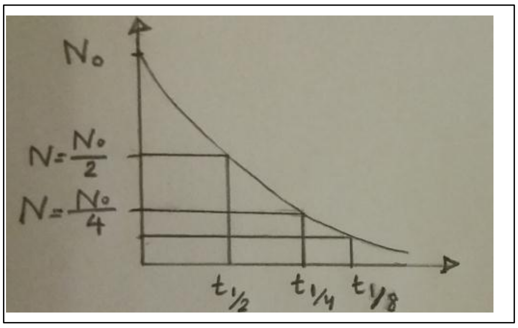

Fig. 1

Q7: How can we relate τ to λ? Derive a formula for this relationship.

Answer: We put $R=R_0e^{-1}$ in Eq. (5) and substitute τ for t. Solving for τ we get:

$$R_0e^{-1}=R_0e^{-\lambda\tau} \quad\rightarrow\quad e^{-1}=e^{-\lambda\tau} \quad\rightarrow\quad 1=\lambda\tau \quad\rightarrow\quad \therefore\ \tau=\frac{1}{\lambda}$$

Q8: How can we relate $t_{1/2}$ to τ? Derive a formula for this relationship.

Answer: From the result of Q7, we have: $\tau=\frac{1}{\lambda}$ ………(a).

Multiplying both sides of Eq. (a) by $\ln 2$ we obtain: $\tau\ln 2=\frac{\ln 2}{\lambda}$ ………(b).

From the result of Q6, we have: $t_{1/2}=\frac{\ln 2}{\lambda}$ ………(c).

Equating Eq's. (b) and (c), we get: $t_{1/2}=\frac{\ln 2}{\lambda}=\tau\ln 2 \quad\rightarrow\quad \therefore\ t_{1/2}=\tau\ln 2$ ………(7).

2. Alpha decay

3. Most nuclides with $A > 150$ undergo alpha decay. If a nucleus undergoes alpha decay, it transforms to a different nuclide by emitting an alpha particle, i.e.,

$${}^{A}_{Z}P_N \rightarrow {}^{A-4}_{Z-2}D_{N-2} + \alpha\,(\equiv {}^{4}\text{He}) + Q_\alpha \;\; .... \;(8)$$

where ${}^{A}_{Z}P_N$ is the parent nucleus, ${}^{A-4}_{Z-2}D_{N-2}$ is the daughter nucleus and Q_α is the $\alpha-$disintegration energy (or $\alpha-$decay energy). For example, when uranium ${}^{238}\text{U}$ undergoes alpha decay, it transforms to thorium ${}^{234}\text{Th}$, i.e., ${}^{238}\text{U} \rightarrow {}^{234}\text{Th} + \alpha\,(\equiv {}^{4}\text{He}) + Q_\alpha$ .... (9)

4. **Note that** the conservation of momentum and energy requires the following:

$$P_P = P_D + P_\alpha \quad \rightarrow \quad 0 = P_D + P_\alpha \quad \rightarrow \quad \therefore P_D = P_\alpha . \;\; .... \;(10)$$

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha \quad \rightarrow \quad M_P c^2 = (M_D + M_\alpha)c^2 + (T_D + T_\alpha)$$

$$\rightarrow \quad M_P c^2 = (M_D + M_\alpha)c^2 + Q_\alpha \quad \rightarrow \quad \therefore Q_\alpha = \{M_P - (M_D + M_\alpha)\}c^2 \;.......... \;(11)$$

Here M_P, M_D and M_α are nuclear masses (in atomic mass unit = u) of parent nucleus, daughter nucleus, and alpha particle.

T_D and P_D are the kinetic energy and momentum of daughter nuclei.

T_α and P_α are the kinetic energy and momentum of alpha particle.

P_P is the momentum of parent nucleus.

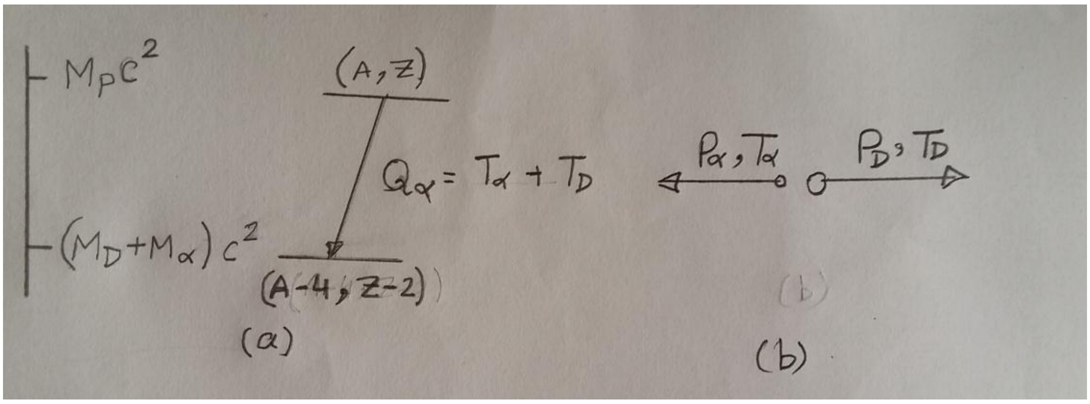

Fig.2: Alpha decay of a nucleus (a) Energy diagram, (b) Momentum diagram (see Meyerhof book).

5. Eq. (10) can be also rewritten as $Q_\alpha = 931.5\{M_P(A,Z) - M_D(A-4, Z-2) - M_\alpha\}$.......... (12)

6. Alpha decay can occur spontaneously (without an external source of energy) because the total mass of the decay products (i.e., the daughter nucleus and alpha particle) is less than the mass of the original nuclide (i.e., the parent nucleus). Thus the total mass energy of the decay products is less than the mass energy of the original nuclide.

7. The decay energy Q_α can be determined either by Eq. (12) or by $Q_\alpha = T_D + T_\alpha$ .... (13)

   Thus Q_α is defined as the sum of the resultant kinetic energies T_D and T_α (see Eq. (13).

8. The kinetic energies T_D and T_α are small enough so that nonrelativistic expressions may be used to evaluate them, i.e., $T_D = \dfrac{P_D^2}{2M_D} \quad \rightarrow \quad T_D = \dfrac{P_\alpha^2}{2M_D} \quad \rightarrow \quad T_D = \dfrac{(M_\alpha V_\alpha)^2}{2M_D}$

$$\rightarrow \quad T_D = \frac{M_\alpha}{M_D}\frac{M_\alpha V_\alpha^2}{2} \quad \rightarrow \quad \therefore\ T_D = \frac{M_\alpha}{M_D}T_\alpha \ \text{..........(14)}$$

Use Eq. (14) in Eq. (13), we get: $Q_\alpha = \frac{M_\alpha}{M_D}T_\alpha + T_\alpha \quad \rightarrow \quad Q_\alpha = \left(\frac{M_\alpha}{M_D} + 1\right)T_\alpha$

$$\rightarrow \quad Q_\alpha = \left(\frac{M_\alpha + M_D}{M_D}\right)T_\alpha \quad \rightarrow \quad \therefore\ Q_\alpha \approx \frac{A}{A-4}T_\alpha \ \text{..........(15)}$$

where A is the mass number of the parent. The alpha particle kinetic energy T_α is always less than the decay energy Q_α. It is clear from the above consideration that alpha decay cannot take place unless Q_α is positive.

9. Referring to the definition of alpha separation energy S_α (see lecture notes of chapter 2), we see from Eq. (12) that $Q_\alpha = 931.5\{M_P(A,Z) - M_D(A-4, Z-2) - M_\alpha\} \rightarrow \therefore\ Q_\alpha = -S_\alpha$ .... (16)
10. Referring to Eq. (16), Q_α can be related to the binding energies of the nuclei as
$Q_\alpha = B_D(A-4, Z-2) + B_\alpha(4,2) - B_P(A,Z)$ .... (17)
11. From Eq. (17) we can see that whenever the daughter nucleus $(A-4, Z-2)$ is magic (i.e., it has a large binding energy), the alpha decay energy is particularly high. Conversely, whenever the parent nucleus (A,Z) is magic, the alpha decay energy is particularly low.

<u>Solved problems</u>:

Q9: If the atomic masses for ^{238}U = 238.05079 u, ^{234}Th = 234.04363 u, and ^{4}He = 4.00260 u. Calculate the energy released during the alpha decay of ^{238}U.

Answer: The decay process is given by: $^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$.

Using the given atomic masses in Eq. (12), we get:

$$Q_\alpha = 931.5\{M_P(A,Z) - M_D(A-4, Z-2) - M_\alpha\}$$

$$\rightarrow \quad Q_\alpha = 931.5\{238.05079\,u - 234.04363\,u - 4.00260\,u\} = 931.5\{0.00456\ u\} = 4.25 \text{ MeV.}$$

Q10: If the atomic masses for ^{238}U = 238.05079 u, ^{237}Pa = 237.05121 u, and ^{1}H = 1.00783 u. Show that ^{238}U cannot spontaneously emit a proton. (Pa is the symbol for the element protactinium with Z=91)

Answer: If this happened, the decay process is: $^{238}_{92}\text{U} \rightarrow {}^{237}_{91}\text{Pa} + {}^{1}_{1}\text{H}$.

Using the given atomic masses in the following Equation, we get:

$$Q_\alpha = 931.5\{M_P(A,Z) - M_D(A-1, Z-1) - m_P(1,1)\}, \text{ where } m_P(1,1) \text{ is the atomic mass of the proton}$$

$$\rightarrow \quad Q_\alpha = 931.5\{238.05079\,u - 237.05121\,u - 1.00783\,u\} = -931.5\{0.00825\ u\} = -7.68 \text{ MeV.}$$

It is clear that the mass of the two decay products (237.05121 u + 1.00783 u) is larger than the mass of ^{238}U by $\Delta m = 0.00825\,u$, with decay energy $Q_\alpha = -7.68$ MeV. The minus sign indicates that we must add 7.68 MeV to a ^{238}U nucleus before it will emit a proton. Thus the nucleus ^{238}U cannot spontaneously emit a proton.

ملاحظة: اذا كانت Q_α موجبة فأن النواة الأم يمكن ان تكون باعثة لجسيمات الفا بشكل تلقائي. اما اذا كانت Q_α سالبة فأن النواة الأم لايمكن ان تكون باعثة لجسيمات الفا بشكل تلقائي.

Q11: If the atomic masses for ^{224}Ra = 224.020217 u, ^{220}Rn = 220.011014, and ^{4}He = 4.002603 u. Find the kinetic energies T_α and T_D during the alpha decay of ^{224}Ra. (Here Ra ≡ Radium and Rn ≡ Radon)

Answer: The decay process is given by: $^{224}_{88}\text{Ra} \rightarrow ^{220}_{86}\text{Rn} + ^{4}_{2}\text{He}$.

Using the given atomic masses in Eq. (12), we get:

$$Q_\alpha = 931.5\{M_P(A,Z) - M_D(A-4, Z-2) - M_\alpha\}$$

$$\rightarrow Q_\alpha = 931.5\{224.020217\,u - 220.011014\,u - 4.002603\,u\} = 6.148 \text{ MeV.}$$

Using Eq. (15), we get:

$$Q_\alpha = \frac{A}{A-4}T_\alpha \rightarrow T_\alpha = \frac{A-4}{A}Q_\alpha \rightarrow T_\alpha = \frac{224-4}{224}\times 6.148 \rightarrow \therefore T_\alpha = 6.038 \text{ MeV.}$$

Using Eq. (15), we get: $Q_\alpha = T_D + T_\alpha \rightarrow T_D = Q_\alpha - T_\alpha \rightarrow \therefore T_D = 6.148 - 6.038 \rightarrow T_D = 0.11$ MeV.

3. Beta decay

12. The beta decay is a radioactive decay in which a proton in a nucleus is converted into a neutron (or vice-versa). Thus A is constant, but Z and N change by 1. In the process the nucleus emits a beta particle (either an electron or a positron) and the neutrino, which is a virtual massless particle.
13. Mostly nuclei with an excess of neutrons will decay by β^- while nuclei with an excess of protons will either decay by β^+ or if surrounded by atomic electrons (such as, k-shell electron), decay by electron capture.

3.1. Types of beta decay

There are 3 types of beta decay:

a) β^- decay (or negative beta decay): $^{A}_{Z}X_N \rightarrow ^{A}_{Z+1}Y_{N-1} + \beta^- + \bar{v}$.

The underlying reaction for β^- decay is $n \rightarrow p + \beta^- + \bar{v}$ that corresponds to the conversion of a neutron into a proton with the emission of β^- and an anti-neutrino ($\bar{v}$).

Examples: $^{14}_{6}C_8 \rightarrow ^{14}_{7}N_7 + \beta^- + \bar{v}$, $^{6}_{2}He_4 \rightarrow ^{6}_{3}Li_3 + \beta^- + \bar{v}$, $^{64}_{29}Cu_{35} \rightarrow ^{64}_{30}Zn_{34} + \beta^- + \bar{v}$.

b) β^+ decay (or positive beta decay): $^{A}_{Z}X_N \rightarrow ^{A}_{Z-1}Y_{N+1} + \beta^+ + v$.

The underlying reaction for β^+ decay is $p \rightarrow n + \beta^+ + v$ that corresponds to the conversion of a proton into a neutron with the emission of β^+ and a neutrino (v).

Examples: $^{10}_{6}C_4 \rightarrow ^{10}_{5}B_5 + \beta^+ + v$, $^{14}_{8}O_6 \rightarrow ^{14}_{7}N_7 + \beta^+ + v$, $^{13}_{7}N_6 \rightarrow ^{13}_{6}C_7 + \beta^+ + v$.

c) electron capture: $^{A}_{Z}X_N + e^- \rightarrow ^{A}_{Z-1}Y_{N+1} + v$.

The underlying reaction for electron capture is $p + e^- \rightarrow n + v$ that corresponds to the conversion of a proton into a neutron with the emission of a neutrino (v).

Examples: ${}^{10}_{6}C_4 + e^- \rightarrow {}^{10}_{5}B_5 + \nu$, ${}^{14}_{8}O_6 + e^- \rightarrow {}^{14}_{7}N_7 + \nu$, ${}^{13}_{7}N_6 + e^- \rightarrow {}^{13}_{6}C_7 + \nu$.

3.2. Neutrino hypothesis

All difficulties concerning the conservation laws were overcome by the neutrino hypothesis of Pauli (1933). He proposed that another particle, besides the electron/positron ($\beta^{\pm}$), is emitted in beta decay. To this particle (i.e., the neutrino), Pauli assigned zero charge, zero or nearly zero mass ($m_\nu = m_e/2000$), and an intrinsic spin angular momentum $1/2\ \hbar$. It carries off energy and linear momentum. It has nearly the same velocity of light. It does not interact with matter.

3.3. Conservation laws

a) Energy conservation: As the neutrino is hard to detect, initially the beta decay seemed to violate energy conservation. Introduction an extra particle (the neutrino) in the process allows one to respect conservation of energy. The kinetic energy (Q_β) is shared by the neutrino and the electron/positron ($\beta^{\pm}$). Then, the $\beta^{\pm}$ (remember, the only particle that we can really observe) does not have a fixed energy, as it was for the gamma photon. Thus it will exhibit continues spectrum of energy as well as a distribution of momenta. Note that the difference between the spectra of the $\beta^{\pm}$ particles is due to the Coulomb repulsion or attraction from the nucleus.

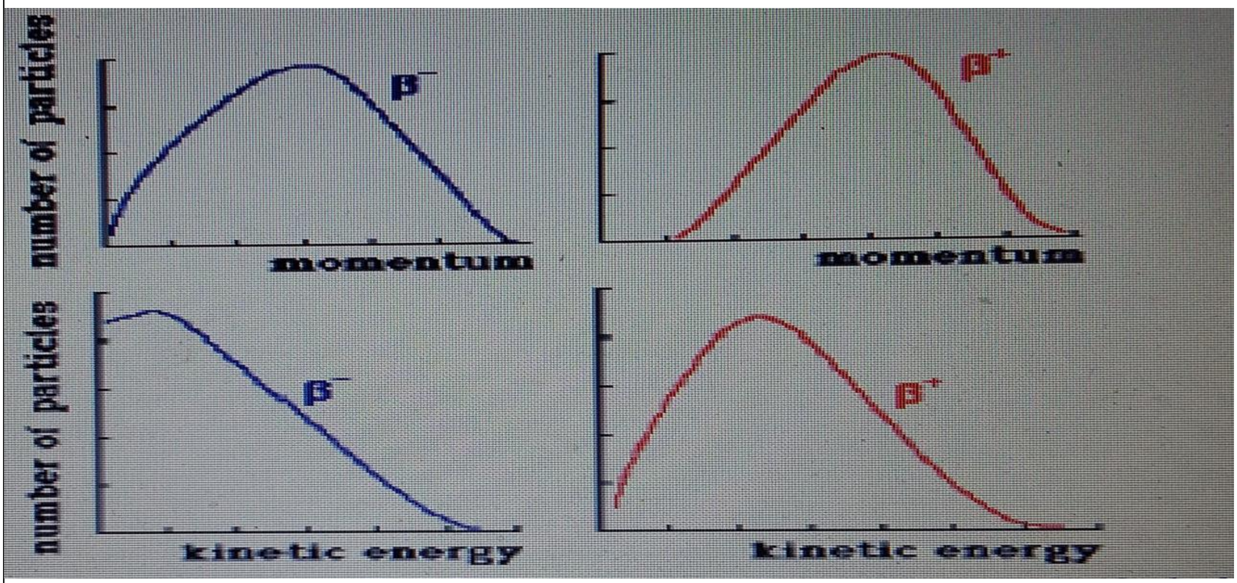

Fig. 3: Beta decay spectra: Distribution of momentum (top plots) and kinetic energy (bottom) for β^- (left) and β^+ (right) decay.

b) Momentum conservation: The momentum is also shared between the electron/positron ($\beta^{\pm}$) and the neutrino. Thus the observed ($\beta^{\pm}$) momentum distribution ranges from zero to a maximum possible momentum transfer.

Q12: How does the linear momentum remain conserved by the following beta decay $^{13}_{7}N_6 \to ^{13}_{6}C_7 + \beta^+ + v$? Explain in brief.

الجواب: ان نواتج تحلل $^{13}_{7}N_6$ الساكنة هي $^{13}_{6}C_7$ و β^+ (انظر الى Fig. 4). من الواضح ان محصلة الزخم الخطي للجسيمتين $^{13}_{6}C_7$ و β^+ لاتساوي صفر. في حين يجب ان تساوي صفر لأن $^{13}_{7}N_6$ قد تحللت وهي ساكنة مما يعني ان زخمها الخطي قبل التحلل كان صفر. ولكي يبقى الزخم الخطي محفوظا لابد من انبعاث جسيمة ثالثة زخمها الخطي يعادل ويعاكس محصلة زخمي $^{13}_{6}C_7$ و β^+ وكما موضح في الشكل (Fig. 4).

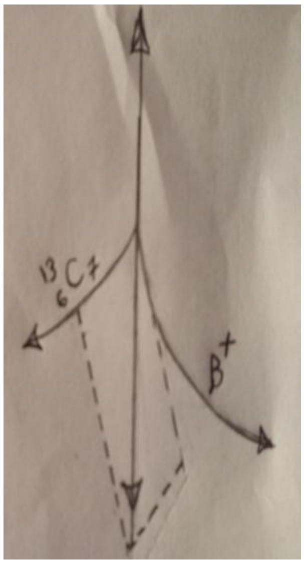

Fig. 4

c) Angular momentum conservation: Both the electron/positron ($\beta^{\pm}$) and the neutrino have spin 1/2.

Q13: How does the angular momentum remain conserved by the beta decay? Explain in brief.

الجواب: انه معروف جيدا بأن الزخم الزاوي لكل من الالكترون والبروتون والنيترون يساوي 1/2 (S_e=S_p=S_n=1/2). لذا فأن الزخم الزاوي في المعادلتين $p \to n + \beta^+ + v,$ or $n \to p + \beta^- + \bar{v}$ سيكون غير محفوظ حيث ان الزخم الزاوي قبل عملية تحلل بيتا لايساوي الزخم الزاوي بعد عملية التحلل. اي ان:

$$p \to n + \beta^+ \quad \to \quad 1/\vec{2} \neq 1/\vec{2} + 1/\vec{2} \quad \to 1/2 = 0, 1$$

$$or \quad n \to p + \beta^- \quad \to \quad 1/\vec{2} \neq 1/\vec{2} + 1/\vec{2} \quad \to 1/2 = 0, 1$$

ولكي يكون الزخم الزاوي لعملية تحلل بيتا محفوظ لذا يجب ان تنبعث جسيمة ثالثة تسمى النيترينو (v) او ضديدها ($\bar{v}$) والتي يفترض ان يكون زخمها الزاوي يساوي 1/2.

3.4. Energy of Beta decay (Q_β)

a) Energy of β^- decay (Q_{β^-}): It is the sum of the kinetic energies T_D, T_{β^-} and $T_{\bar{v}}$, i.e., $Q_{\beta^-} = T_D + T_{\beta^-} + T_{\bar{v}}$. But $T_D \approx 0$ because M_D is very large, thus $Q_{\beta^-} \approx T_{\beta^-} + T_{\bar{v}}$ ....(18)

It is also given by the mass difference of isobars, i.e.,

$$Q_{\beta^-} = 931.5\{M_P(A, Z) - M_D(A, Z+1)\} = T_{\beta^-} + T_{\bar{v}} \quad ....(19)$$

Note that M_P and M_D are in atomic mass unit (u).

b) Energy of β^+ decay (Q_{β^+}): Similarly $Q_{\beta^+} = T_D + T_{\beta^+} + T_v \;\to\; \therefore\; Q_{\beta^+} \approx T_{\beta^+} + T_v$ ....(20)

And we can also write: $Q_{\beta^+} = 931.5\{M_P(A, Z) - M_D(A, Z-1) - 2m_e\} = T_{\beta^+} + T_v$ ....(21)

Note: In Eqs (19) and (21), M_P and M_D are in atomic mass unit (u).

c) Energy of Electron capture ($Q_{e.c}$): It is the sum of the kinetic energies T_D and $T_{\bar{v}}$, i.e.,

$$Q_{e.c} = T_D + T_v \quad \to \quad \therefore\; Q_{e.c} \approx T_v \quad .... \;(22)$$

And also given by $Q_{e.c} = 931.5\{M_P(A, Z) - M_D(A, Z-1)\} = T_v$ .... (23)

Subtracting Eq. (21) from Eq. (23), we get:

$$Q_{e.c} - Q_{\beta^+} = 931.5 \times (2m_e) \rightarrow Q_{e.c} = Q_{\beta^+} + 931.5 \times (2m_e) \rightarrow \therefore Q_{e.c} = Q_{\beta^+} + 1.022\,\text{MeV} \ldots\ldots(24)$$

Note: Eq. (24) gives the conclusion that the energy released in electron capture is larger than that in β^+ decay, i.e., $Q_{e.c} > Q_{\beta^+}$.

Q14. If the atomic masses for ${}^{64}_{29}Cu_{35}$ = 63.929759 u, ${}^{64}_{30}Zn_{34}$ = 63.929145, and ${}^{64}_{28}Ni_{36}$ = 63.927958. Show that the parent nucleus ${}^{64}_{29}Cu_{35}$ can decay by β^-, β^+ and electron capture, i.e., It decays, respectively, by: ${}^{64}_{29}Cu_{35} \rightarrow {}^{64}_{30}Zn_{34} + \beta^- + \bar{\nu}$, ${}^{64}_{29}Cu_{35} \rightarrow {}^{64}_{28}Ni_{36} + \beta^+ + \nu$, and ${}^{64}_{29}Cu_{35} + e^- \rightarrow {}^{64}_{28}Ni_{36} + \nu$.

Answer: u

Using Eqs (19) we get: $Q_{\beta^-} = 931.5\{63.929759\,\text{u} - 63.929145\,\text{u}\} = 0.573\ \text{MeV}$

Using Eqs (21) we get: $Q_{\beta+} = 931.5\{63.929759\ \text{u} - 63.927958\ \text{u} - 2m_e\} = 0.656\ \text{MeV}$

Using Eqs (23) we get: $Q_{e.c} = 931.5\{63.929759\ \text{u} - 63.927958\ \text{u}\} = 1.1.678\ \text{MeV}$

As the decay energies Q_{β^-}, Q_{β^+}, and $Q_{e.c}$ have positive value, thus ${}^{64}_{29}Cu_{35}$ can decay by the three methods.

3.4. Classification of beta decay:

It is well known that both electron and neutrino have intrinsic spin $\vec{s}_\beta = \vec{s}_\nu = \frac{1}{2}\hbar$. Depending on the total spin angular momentum $\vec{S}_\beta = \vec{s}_\beta + \vec{s}_\nu$ of electron-neutrino pair; there are two types of classifications:

a) Fermi decay: The electron-neutrino pair has a total spin $\vec{S}_\beta = 0$. Here $\vec{s}_\beta$ and $\vec{s}_\nu$ are in opposite directions.

b) Gamow-Teller decay: The electron-neutrino pair has a total spin $\vec{S}_\beta = 1$. Here $\vec{s}_\beta$ and $\vec{s}_\nu$ are in the same direction.

Note: If the total orbital angular momentum of electron-neutrino pair is $\vec{L}_\beta$, then we have:

- Decays with $\vec{L}_\beta = 0$ are called allowed.
- Decays with $\vec{L}_\beta = 1$ are called first forbidden.
- Decays with $\vec{L}_\beta = 1$ are called second forbidden, etc.

3.4.1. Selection rules for beta decay

a) Conservation of angular momentum:

$$\vec{I}_P = \vec{I}_D + \vec{S}_\beta + \vec{L}_\beta \rightarrow \vec{I}_P - \vec{I}_D = \vec{S}_\beta + \vec{L}_\beta \rightarrow \therefore \Delta I = \vec{S}_\beta + \vec{L}_\beta \ldots\ldots\ldots\ldots (25)$$

b) Conservation of Parity:

$$\pi_P . \pi_D = (-1)^{L_\beta} \ldots\ldots\ldots\ \ldots\ldots\ldots\ (26)$$

3.4.2. Outline for classification beta decays

a) Determine $\vec{L}_\beta$ by Eq. (26).

Eg. 1: If $\pi_P = +$ and $\pi_D = -$. **Answer:** $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow +.- = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 1, 3, \ldots$

Eg. 2: If $\pi_P = -$ and $\pi_D = +$. **Answer:** $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow -.+ = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 1, 3, \ldots$

Eg. 3: If $\pi_P = +$ and $\pi_D = +$. **Answer:** $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow +.+ = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 0, 2, \ldots$

Eg. 4: If $\pi_P = -$ and $\pi_D = -$. **Answer:** $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow -.- = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 0, 2, \ldots$

b) Determine ΔI by the vector addition rule: $\left|\vec{I}_P - \vec{I}_D\right| \le \Delta I \le \vec{I}_P + \vec{I}_D$ ……….. (27)

Eg. 1: If $\vec{I}_P = \vec{3}$ and $\vec{I}_D = \vec{2}$. **Answer:** Use the rule of Eq. (27), we can get: $\Delta I = \vec{1}, \vec{2}, \vec{3}, \vec{4}, \vec{5}$.

Eg. 2: If $\vec{I}_P = \frac{\vec{1}}{2}$ and $\vec{I}_D = \frac{\vec{1}}{2}$. **Answer:** Use the rule of Eq. (27), we can get: $\Delta I = \vec{0}, \vec{1}$.

c) Determine $\vec{S}_\beta$ by substituting the obtained values of ΔI and $\vec{L}_\beta$ in Eq. (25). To determine the predominant decay mode (i.e. the more probability decay), we must take the smallest value of $\vec{L}_\beta$.

3.4. 3. Solved problems

Q15. Classify the following beta decay: ${}^6_2He_4$ $(I_P = 0^+) \rightarrow {}^6_3Li_3(I_D = 1^+) + ...$, and hence find the predominant decay mode.

Answer: It is clear that the neutron number in the initial nucleus (${}^6_2He_4$) is larger by one than that of the final nucleus (${}^6_3Li_3$), i.e., we have the decay: $n \rightarrow p + \beta^- + \bar{v}$. Thus ${}^6_2He_4$ decays by β^- as: ${}^6_2He_4 \rightarrow {}^6_3Li_3 + \beta^- + \bar{v}$.

1) From the transition $0^+ \rightarrow 1^+$: We have $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow +.+ = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 0, 2, \ldots$
2) From the rule $\left|\vec{I}_P - \vec{I}_D\right| \le \Delta I \le \vec{I}_P + \vec{I}_D$: We can get $\Delta I = \vec{1}$.
3) Substitute $\vec{L}_\beta = 0$ (allowed) and $\Delta I = \vec{1}$ in $\Delta I = \vec{S}_\beta + \vec{L}_\beta$, we get: $\vec{1} = \vec{S}_\beta + \vec{0} \rightarrow \therefore \vec{S}_\beta = \vec{1}$ (i.e., Gamow Teller decay).
   $\therefore$ The predominant decay mode is allowed, Gamow Teller decay, β^- decay.

Q16. Classify the following beta decay: ${}^{14}_8O_6(0^+) \rightarrow {}^{14}_7N_7(0^+) + ...$, and hence find the predominant decay mode.

Answer: It is clear that the proton number in the initial nucleus (${}^{14}_8O_6$) is larger by one than that of the final nucleus (${}^{14}_7N_7$), i.e., We have the decay: $p \rightarrow n + \beta^+ + v$. Thus ${}^{14}_8O_6$ decays by β^+ as: ${}^{14}_8O_6(0^+) \rightarrow {}^{14}_7N_7(0^+) + \beta^+ + v$.

1) From the transition $0^+ \rightarrow 0^+$: We have $\pi_P.\pi_D = (-1)^{L_\beta} \rightarrow +.+ = (-1)^{L_\beta} \rightarrow \therefore L_\beta = 0, 2, \ldots$
2) From the rule $\left|\vec{I}_P - \vec{I}_D\right| \le \Delta I \le \vec{I}_P + \vec{I}_D$: We can get $\Delta I = \vec{0}$.
3) Substitute $\vec{L}_\beta = 0$ (allowed) and $\Delta I = \vec{0}$ in $\Delta I = \vec{S}_\beta + \vec{L}_\beta$, we get: $\vec{0} = \vec{S}_\beta + \vec{0} \rightarrow \therefore \vec{S}_\beta = \vec{0}$ (i.e., Fermi decay).

$\therefore$ The predominant decay mode is allowed, Fermi decay, β^+ decay.

Q17. Classify the following beta decays:
$^{10}_{4}Be_6(3^+) \rightarrow ^{10}_{5}B_5(0^+) + ...,$
$^{35}_{16}S_{19}(3/2^+) \rightarrow ^{35}_{17}Cl_{18}(3/2^+) + ...,$
$^{91}_{39}Y_{52}(1/2^-) \rightarrow ^{91}_{40}Sr_{51}(5/2^+) + ...,$
$^{38}_{17}Cl_{21}(2^-) \rightarrow ^{38}_{18}Ar_{20}(2^+) + ...,$
and hence find the predominant decay modes.

4. Gamma decay

14. If the initial excited (the parent) nucleus has a mass $M^*_{i/P}(A, Z)$ and the final state (the daughter) nucleus has a mass $M_{f/D}(A, Z)$. The conservation of energy and momentum require that
$M^*_{i/P}(A, Z)c^2 = M_{f/D}(A, Z)c^2 + E_\gamma + T_{f/D}$ ....(28)
and $0 = \vec{P}_\gamma + \vec{P}_{f/D}$ ....(29)

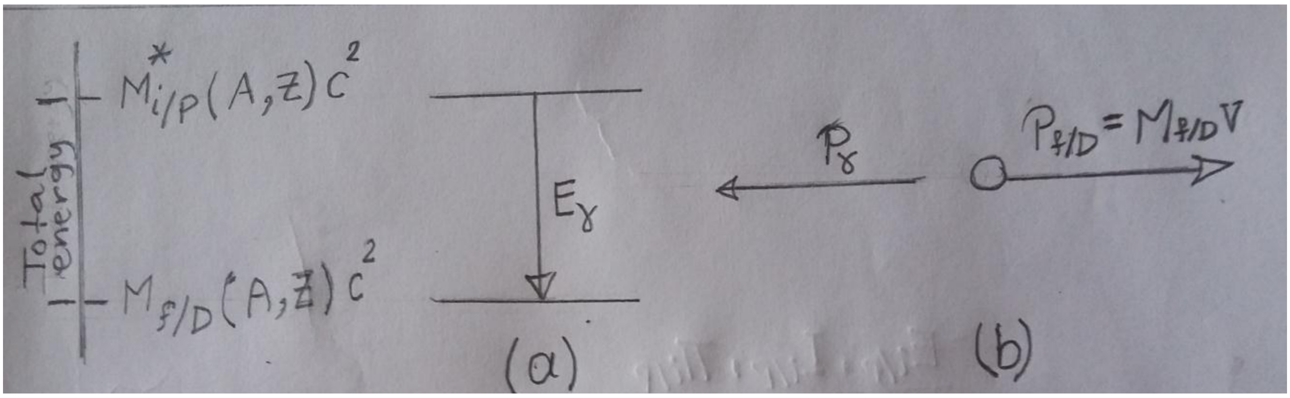

Fig. 5: Gamma decay of a nucleus (a) Energy diagram (b) Momentum diagram.

where E_γ and $\vec{P}_\gamma$ are the energy and momentum of gamma ray, $T_{f/D}$ and $\vec{P}_{f/D}$ are the kinetic energy and momentum of the daughter nucleus. $T_{f/D}$ is negligible because it is very small compared with E_γ.

15. Gamma decay energy (Q_γ) is defined as $Q_\gamma = 931.5\{M^*_{i/P}(A, Z) - M_{f/D}(A, Z)\} = E_\gamma$ ....(30)

16. Gamma ray has a discrete energy spectrum.

4. 1. Selection rules for gamma decay

a) Conservation of energy requires
$Q_\gamma = E_{i/P} - E_{f/D} = 931.5\{M^*_{i/P}(A, Z) - M_{f/D}(A, Z)\} = E_\gamma$ .... (31)
where $E_{i/P}$ is the energy of initial excited (the parent) nucleus,
and $E_{f/D}$ is the energy of final state (the daughter) nucleus.

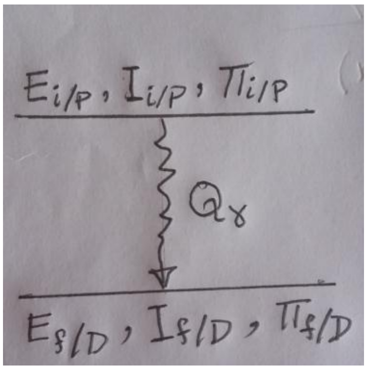

Fig. 6

b) Conservation of angular momentum requires

$\vec{L} = \vec{I}_{i/P} + \vec{I}_{f/D} \quad \rightarrow \quad \therefore \left| I_{i/P} - I_{f/D} \right| \leq L \leq I_{i/P} + I_{f/D}$

or $\quad L = I_{i/P} - I_{f/D},\ I_{i/P} - I_{f/D} + 1, \ldots\ I_{i/P} + I_{f/D}$ .... (32)

where $\vec{L}$ is the multipolarity of gamma ray, $\vec{I}_{i/P}$ and $\vec{I}_{f/D}$ are the total angular momenta of initial state (parent nucleus) and finial state (daughter nucleus), respectively.

c) Conservation of parity requires:

$\pi_{i/P}.\pi_{f/D} = (-1)^L$ for electric multipole radiation ....(33)

$\pi_{i/P}.\pi_{f/D} = (-1)^{L+1}$ for magnetic multipole radiation ....(34)

Notes:

1. The Number of pole is given by 2^L.
2. For a given multipolarity (L), the probability of emitting an electric radiation (EL) is larger than that of magnetic radiation (EL), i.e., $EL > ML$.
3. We have also: $E1 > M1 > E2 > M2 > E3 > M3 > E4 > M4 \ldots..$ ....(35)
4. $E1$ is the electric dipole, $M1$ is the magnetic dipole, $E2$ is the electric quadrupole, $M2$ is the magnetic quadrupole, $E3$ is the electric octopole, $M2$ is the magnetic octopole, $E4$ is the electric 16-pole, and $M4$ is the magnetic 16-pole….. etc.
5. No gamma decay for the transition $0^+ \rightarrow 0^+$: Because Eq. (32) gives only $L = 0$. Thus the number of poles = $2^L = 2^0 = 1.$, i.e., there is no electromagnetic multipole moments in which it follows that there is no emitted gamma radiation.

4. 2. Solved problems

Q18. Classify the possible multipole radiations of the gamma decay (or gamma transition): $9/2^+ \rightarrow 1/2^-$, and hence find the predominant decay mode.

Answer:

1) Use the selection rule of Eq. (32) we get: $\left|9/2 - 1/2\right| \leq L \leq 9/2 + 1/2 \quad \rightarrow \quad \therefore L = 4, 5.$
2) Eq. (33) is applicable only for $L = 5$, i.e., $+ .- = (-1)^5 \rightarrow \therefore$ the sign of LHS = the sign of RHS. Thus the gamma decay (gamma transition) is electric of type $E5$.
3) Similarly, Eq. (34) is applicable only for $L = 4$, i.e., $+ .- = (-1)^{4+1} \rightarrow \therefore$ the sign of LHS = the sign of RHS. Thus the gamma decay (gamma transition) is Magnetic of type $M4$.
4) The possible multipole radiations of the gamma decay are $M4$ and $E5$.
5) With the help of Eq. (35), the predominant decay mode is $M4$.

Q19. Classify the possible multipole radiation of the gamma transition: $3^+ \rightarrow 1^+$, and hence find the predominant decay mode.

Answer: We can repeat the steps mentioned in Q18 and find the following:

1) From Eq. (32) we get: $\left|3 - 1\right| \leq L \leq 3 + 1 \quad \rightarrow \quad \therefore L = 2, 3, 4.$
2) Eq. (33) is applicable only for $L = 2, 4.$ Thus we get electric transition of type $E2$ and $E4$.
3) Eq. (34) is applicable only for $L = 3.$ Thus we get magnetic transition of type $M3$.
4) The possible multipole radiations of the gamma transition are $E2$, $M3$, and $E4$.
5) With the help of Eq. (35), the predominant decay mode is $E2$.

Q20. Classify the following gamma decays:

$(a)\ 2^+ \to 0^+,\quad (b)\ 1^+ \to 0^+,\quad (c)\ 5/2^- \to 1/2^+,\quad (d)\ 2^+ \to 1^-,\quad (e)\ 0^+ \to 0^+$ and hence find the predominant decay modes.

4. Internal conversion

17. It is defined as the process of ejecting the k or l atomic electron when gamma photon, emitted by an excited nucleus, is absorbed by that atomic electron.

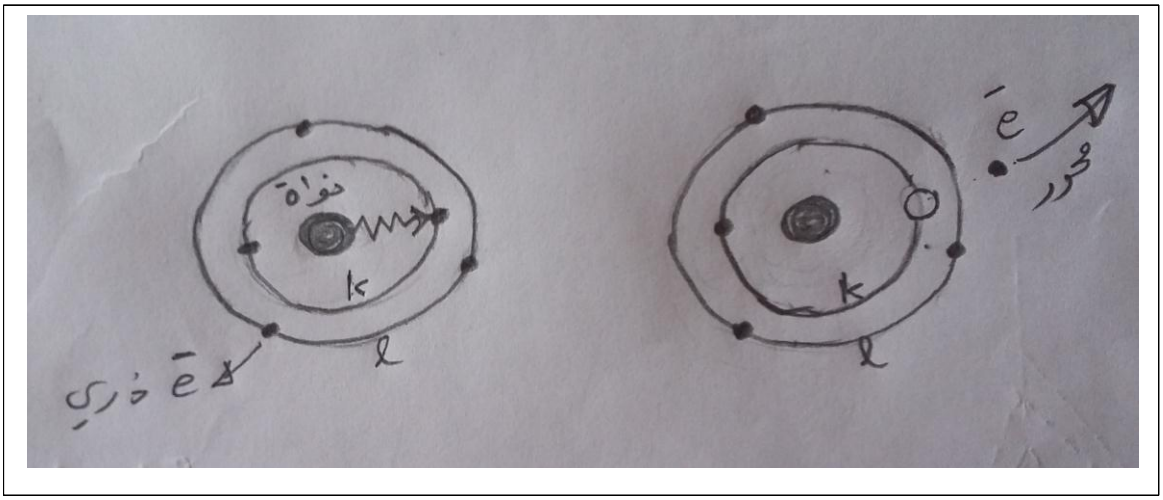

Fig. 7