Representation or Determination of a vector $\vec{v}$

by length and angle :

If $\vec{v}$ is a non-zero vector whose initial point at the origin of $\mathbb{R}^2$ and ϕ is the angle between the positive x-axis and $\vec{v}$ then

$$\vec{v} = (\|\vec{v}\| \cos\phi, \|\vec{v}\| \sin\phi)$$

or $$\vec{v} = (\|\vec{v}\| \cos\phi)\,i + (\|\vec{v}\| \sin\phi)\,j$$

$$= \|\vec{v}\| \left[(\cos\phi)\,i + (\sin\phi)\,j \right]$$

Examples: ① Find the vector of length 2 such that it makes an angle of $\frac{\pi}{4}$ with the positive x-axis.

② Find the angle that the vector $\vec{v}$ makes with the positive x-axis if $\vec{v} = j - \sqrt{3}\,i$

Sol ① $\vec{v} = 2\left(i\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$

$$= \frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\right)i + \frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\right)j = \sqrt{2}\,i + \sqrt{2}\,j$$

$= -\sqrt{3}\,i + j = (-\sqrt{3}, 1)$