Example: Calculate

$(3,5)\cdot(-1,2) = 3(-1)+5(2) = 7$

$(3,2)\cdot(2,-3) = 2(2)+3(-3) = -5$

$(1,-3,4)\cdot(1,5,2) = 1(1)+(-3)5+4(2) = -6$

Theorem (1): If $\vec{u}$ & $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ and m is a scalar ($\in \mathbb{R}$), then:

1. $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$
2. $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$
3. $m(\vec{u}\cdot\vec{v}) = (m\vec{u})\cdot\vec{v} = \vec{u}\cdot(m\vec{v})$
4. $\vec{v}\cdot\vec{v} = \|\vec{v}\|^2$
5. $\vec{0}\cdot\vec{v} = 0$ and $0\vec{v} = \vec{0}$

Angle Between Vectors الزاوية بين المتجهات

Theorem (2): If $\vec{u}$ & $\vec{v}$ are non-zero vectors in $\mathbb{R}^2$ and if θ is the angle between them, then:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$$