Proof: From the law of cosines

$$\|\vec{v}-\vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\,\|\vec{v}\|\cos\theta \quad \ldots \circledast$$

& from properties of dot product in Th.1

$$\|\vec{v}-\vec{u}\|^2 = (\vec{v}-\vec{u})\cdot(\vec{v}-\vec{u})$$

$$= (\vec{v}-\vec{u})\cdot\vec{v} - (\vec{v}-\vec{u})\cdot\vec{u}$$

$$= \vec{v}\cdot\vec{v} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{u}\cdot\vec{u}$$

$$= \|\vec{v}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{u}\|^2$$

By using it in $\circledast$ above ;

$$\cancel{\|\vec{v}\|^2} - 2\vec{u}\cdot\vec{v} + \cancel{\|\vec{u}\|^2} = \cancel{\|\vec{u}\|^2} + \cancel{\|\vec{v}\|^2} - 2\|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

$$\Rightarrow \vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} \qquad \square.$$

Example: Find the angle between the vector $\vec{u} = i - 2j + 2k$ and

a) $\vec{v} = -3i + 6j + 2k$

b) $\vec{w} = 2i + 7j + 6k$ and c) $\vec{z} = -3i + 6j - 6k$