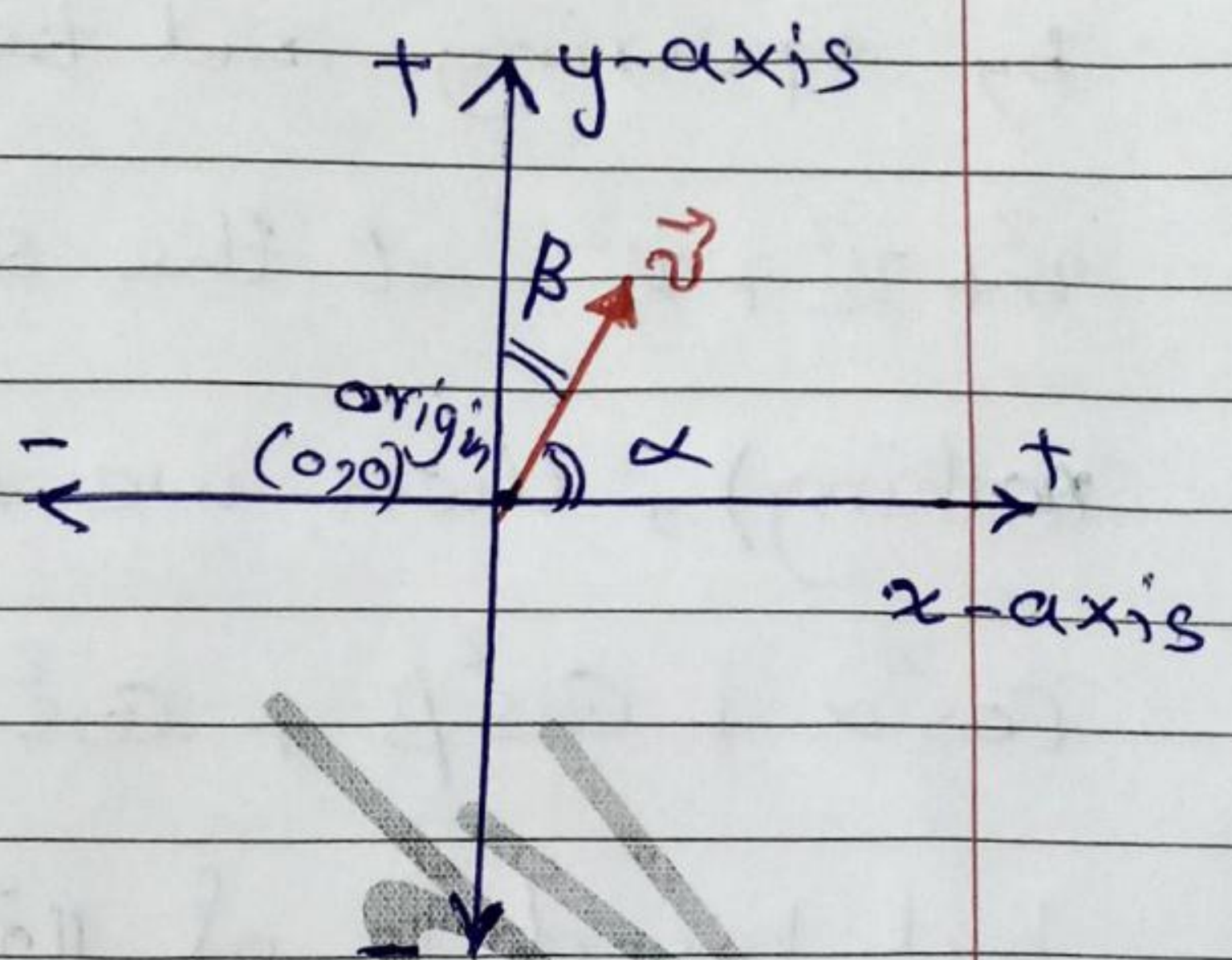


$$\Rightarrow \cos \alpha = \frac{v_1 + 0}{\|\vec{v}\| (1)} = \frac{v_1}{\|\vec{v}\|}$$

$$\& \cos \beta = \frac{v_2}{\|\vec{v}\|}$$



**Theorem:** The direction cosines of a non-zero vector  $\vec{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k} = (v_1, v_2, v_3)$

are:

$$\cos \alpha = \frac{v_1}{\|\vec{v}\|}, \quad \cos \beta = \frac{v_2}{\|\vec{v}\|} \quad \& \quad \cos \gamma = \frac{v_3}{\|\vec{v}\|}$$

then,  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

**Proof:**

$$\begin{aligned} \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= \\ \left(\frac{v_1}{\|\vec{v}\|}\right)^2 + \left(\frac{v_2}{\|\vec{v}\|}\right)^2 + \left(\frac{v_3}{\|\vec{v}\|}\right)^2 &= \\ \frac{v_1^2}{\|\vec{v}\|^2} + \frac{v_2^2}{\|\vec{v}\|^2} + \frac{v_3^2}{\|\vec{v}\|^2} &= \frac{v_1^2 + v_2^2 + v_3^2}{\|\vec{v}\|^2} \end{aligned}$$