by squaring and take the square root for $v_1^2+v_2^2+v_3^2$ at the same time (it is like doing nothing), then we have

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{\left(\sqrt{v_1^2+v_2^2+v_3^2}\right)^2}{\|\vec{v}\|^2}$$

but by def. of $\|\vec{v}\|$

$$=\frac{\|\vec{v}\|^2}{\|\vec{v}\|^2}=1 \quad \square.$$

Example: Find the direction cosines of the vector $\vec{v}=2i-4j+4k$ and approximate the direction angles to the nearest degree.

$(2,-4,4)$

Sol: $\|\vec{v}\|=\sqrt{4+16+16}=\sqrt{36}=6$

then

$$\cos\alpha=\frac{v_1}{\|\vec{v}\|}=\frac{2}{6}=\frac{1}{3}$$

$$\cos\beta=\frac{-2}{3} \quad \text{and} \quad \cos\gamma=\frac{2}{3}$$

$$\therefore \alpha=\cos^{-1}\left(\frac{1}{3}\right)\simeq 71^\circ,\ \beta=\cos^{-1}\left(\frac{-2}{3}\right)=132^\circ$$

$$\text{and} \quad \gamma=\cos^{-1}\left(\frac{2}{3}\right)\simeq 48^\circ$$