Proposition: If $\vec{u}\cdot\vec{v}=0$ and $\vec{u}\neq\vec{0}$, $\vec{v}\neq\vec{0}$ then $\vec{u}\perp\vec{v}$ ($\vec{u}$ & $\vec{v}$ are perpendicular).

Proof:

by Theorem 2 we have:

$$\cos\theta=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$$

and $\vec{u}\cdot\vec{v}=0$ (is given) then

$$\cos\theta=\frac{0}{\|\vec{u}\|\,\|\vec{v}\|}=0$$

$$\therefore\ \theta=\cos^{-1}(0)=\frac{\pi}{2}=90^\circ$$

$\Rightarrow \vec{u}\perp\vec{v}$ or $\vec{u}$ & $\vec{v}$ are perpendicular $\square$.

Cross Product — الضرب الإتجاهي

If $\vec{u}=(u_1,u_2,u_3)$ and $\vec{v}=(v_1,v_2,v_3)$ are vectors in $\mathbb{R}^3$, then the cross product of $\vec{u}$ and $\vec{v}$ respectively which is denoted by $\vec{u}\times\vec{v}$ and defined by:

$$\vec{u}\times\vec{v}=\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}=(-1)^{1+1}(u_2v_3-u_3v_2)\,i$$