(Chapter two) (1)

Numerical Solution of non linear equation

الحلول العددية للمعادلات اللاخطية

(roots finding).

In This chapter we study certain methods of Solving non linear equations in single variable of the form $f(x)=0$, where $f: \mathbb{R} \longrightarrow \mathbb{R}$.

(root) جذر المعادلة هو قيمة المتغير المستقل X الذي يجعل الدالة غير الخطية مساوية الى الصفر

The study of analytical methods for finiding The roots (solutions) of algebraic, for example

There are different ways to find The root s of equation in the form $(f(x)=0)$

1- $p(x) = ax^{2n} + bx^{n} + c = 0$ ~~poly nomal~~ polynomials

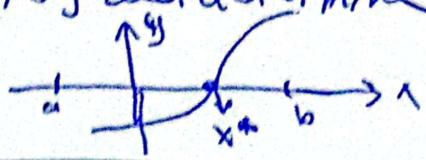
2- $y = ae^{x} + b\cos x + cx = 0$ دوال اسية

3- $y = \sin x + 7x = 0$ trigonometric

(Locating Roots) موقع الجذر

let us suppose That our problem is to find some or all of The roots of The non-linear equation $f(x)=0$, we can locate roots in The following two ways:

@ the roots of The equation $f(x)=0$ are The x-coordinate values of The points of intersection of $f(x)$ with The x-axis. Thus, we may trace the function $f(x)$ in The interval $[a,b]$ and determine graphically The root x^*

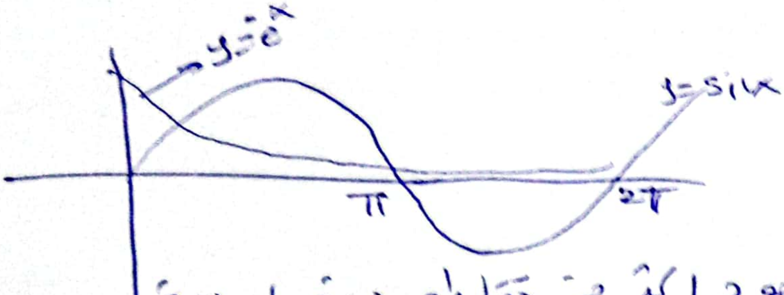


(b) often it is possible to write the function $f(x)$ as the difference of two simple known functions. suppose $f(x) = h(x) - g(x)$. Then, the roots of $f(x)=0$ that is $h(x), g(x)$ are given by the intersection of the curve $h(x)$ and $g(x)$

Ex $e^{x}\sin x - 1 = 0$

$$\sin x = e^{-x}$$
$$h(x) \quad g(x)$$

نرسم الدالتين



الحل الوحيد بين صفر و π لكن تقاطع بين الدالتين الاول في $x=0.6$ والثاني $x=3.1$ تقريبا وهناك عدد لا نهائي من التقاطعات الاخرى عندما تكون قريبة من $n\pi$ $\{n=2,3,\dots\}$

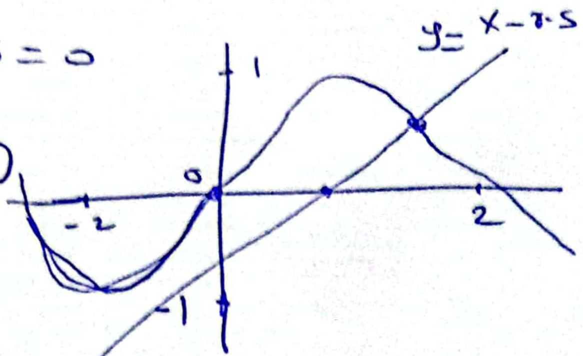
Ex) $\sin x - x + 0.5 = 0$

we put $f(x) = \sin x - x + 0.5 = 0$

suppose $f(x) = h(x) - g(x)$

where $h(x) = \sin x$

$g(x) = x - 0.5$



we deduce from the graph that the equation has one real root near $x = 1.5$

x	1.5	1.49	1.45
$\sin x$	0.9975	0.9967	0.9927
$f(x) = \sin x - x + 0.5$	-0.0025 سالب −	0.067 موجب +	0.0427 +

x	-2	-1.5	-1	-0.5	0
$f(x)$					

We now know that the root lies between 1.49 and 1.50, and we can use a numerical method to obtain a more accurate answer

Ex)

عين مواقع جذور المعادلة

$$f(x)=x^4-7x^3+3x^2+26x-10=0$$

في الفترة $[-8,8]$

اذا اخذنا طول فترة التقسيم $h=4$ فان اشارة الدالة في نقاط التقسيم تكون

x	-8	-4	0	4	8
$f(x)$	$+$	$+$	$-$	$-$	$+$

هنا جذر   هنا جذر

ومن ذلك يتبين وجود جذرين فقط الاول في الفترة $(-4,0)$ والثاني في الفترة $(4,8)$

اما عند اختيار فترة تقسيم اصغر 2 بدلا من 4 فان اشارات الدالة تكون كما يلي

x	-8	-6	-4	-2	0	2	4	6	8
$f(x)$	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$

اي ان هناك جذور في الفترات $(-2,0)$ ، $(0,2)$ ، $(2,4)$ ، $(4,6)$

Bisection Method طريقة التنصيف

This method is based on The repeated appliation of The intermediate value Theorem. suppose f is continuous function on The [a,b], and f(a) & f(b) are of opposite sighs, Then There exists at least one real root of f(x)=0 between a and b

let f be a cont^s function on $[a_0, b_0]$, $f(a_0) < 0$, $f(b_0) > 0$ and $x^* = \frac{a_0 + b_0}{2}$

Then if we calculate $f(x^*)$, which is The function Value at the point of bisection of The interval (a_0, b_0) we will have Three possibilities

(a) $f(x^*) = 0$ in This case x^* is The root of $f(x) = 0$

(b) $f(x^*) < 0$ in This case The root of $f(x) = 0$ lies between x^* and b_0

(c) $f(x^*) > 0$ in This case The root of $f(x) = 0$ lies between a and x^* and a_0

presuming There is just one root, if cas (a) accurs The process is terminated. If either case (b) or (c) occurs, The process of bisection of The interval Containing The root Can be repeated until The root is obtained to The desired accuracy.

Ex Find the root of the following function using bisection method $f(x) = x^3 + 3x - 5 = 0$

Solution

To find interval $f(0) = -5 < 0$

$f(1) = 1 + 3(1) - 5 = -1 < 0$

$f(2) = 8 + 3(2) - 5 = 9 > 0$ $\longrightarrow [1, 2]$

$a = 1, b = 2$

n	a_n	b_n	$P_n = \frac{a+b}{2}$	$f(P_n) = x^3 + 3x - 5$	$\lvert b_n - a_n \rvert$ ϵ
1	1	2	1.5	$2.875 > 0$ موجب	1
2	1	1.5	1.25	$0.703 > 0$ موجب	0.5
3	1	1.25	1.125	$-0.201 < 0$ سالب	0.75
4	1.125	1.25	1.875	$0.237 > 0$	0.125
5	1.125	1.875	1.15625	$0.014 > 0$	0.0625
6	1.125	1.5625	1.1406	$-0.094 < 0$	0.03125
7	1.1406	1.15625	1.1484	$-0.040 < 0$	0.056
8	1.1484	1.15625	1.1523	$-0.0131 < 0$	0.0078
	1.1523	1.5625	الجذر		

After 8 iterations, $P_8 = 1.1523$ approximates the root x^* with an error

$$|x^* - P_8| < |b_9 - a_9| = |1.15625 - 1.1523| = 0.00395 < \epsilon$$

$\epsilon = 0.0078$

$$\text{or } |P_8 - P_7| = |1.1523 - 1.1484| = 0.0039 < \epsilon$$

Theorem: suppose that $f \in C[a,b]$ and $f(a) \cdot f(b) < 0$ ⑥

The Bisection method generates a sequence $\{P_n\}_{n=1}^{\infty}$ approximating a zero p of f with

$$|P_n - P| \leq \frac{b-a}{2^n}, \quad \text{when } n \geq 1$$

Ex) Determine the number of iterations necessary to solve $f(x) = x^3 + 3x - 5 = 0$ with accuracy 10^{-3} Using $a_1 = 1$ and $b_1 = 2$ requires finding an integer n that satisfies:

Solution

صح ايضا

$$|P_n - P| \leq 2^{-n}(\underbrace{b-a}_{2-1}) = 2^{-n} < 10^{-3}$$

$$2^{-n} < 10^{-3} \longrightarrow \log_{10} 2^{-n} < \log_{10} 10^{-3} = \frac{\ln 10^{-3}}{\ln 10} = \frac{-3\cancel{\ln 10}}{\cancel{\ln 10}}$$

$$-n \log_{10} 2 < -3$$

$$n > \frac{3}{\log_{10} 2} \approx 9.96$$

Ex) H.w The equation $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1,2]$