Foundation of Mathematics I

Chapter 1 Logic Theory

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Course Outline
First Semester

Course Title: Foundation of Mathematics (1)

Code subject: MATH103

Instructors: *Mustansiriyah University-College of Science-Department of Mathematics*

Stage: The First

Contents

Chapter 1	Logic Theory	Logic, Truth Table, Tautology, Contradiction, Contingency, Rules of Proof , Logical Implication, Quantifiers, Logical Reasoning, Mathematical Proof.
Chapter 2	Sets	Definitions, Equality of Sets, Set Laws
Chapter 3	Relations on Set	Cartesian Product, Relations.
Chapter 4	Algebra of Mappings	Mappings, Types of Mappings, Composite Mapping.

References

1-Fundamental Concepts of Modern Mathematics. Max D. Larsen. 1970.
2-Introduction to Mathematical Logic, 4th edition. Elliott Mendelson.1997.
3-**اسس الرياضيات, الجزء الاول. تاليف د. هادي جابر مصطفى, رياض شاكر نعوم و نادر جورج منصور. 1980.**
4- A Mathematical Introduction to Logic, 2nd edition. Herbert B. Enderton. 2001.

THE GREEK ALPHABET

letter		*name*	*capital*
α		Alpha	Α
β		Beta	Β
γ		Gamma	Γ
δ		Delta	Δ
ε		Epsilon	Ε
ζ		Zeta	Ζ
η		Eta	Η
θ		Theta	Θ
ι		Iota	Ι
κ		Kappa	Κ
λ		Lambda	Λ
μ		Mu	Μ
ν		Nu	Ν
ξ		Xi	Ξ
ο		Omicron	Ο
π		Pi	Π
ρ		Rho	Ρ
σ	ς	Sigma	Σ
τ		Tau	Τ
υ		Upsilon	Υ
φ		Phi	Φ
χ		Chi	Χ
ψ		Psi	Ψ
ω		Omega	Ω

Chapter One

Logic Theory

1.1. Logic

Definition 1.1.1.

(i) Logic is the theory of systematic reasoning and symbolic logic is the formal theory of logic.

(ii) A **logical proposition (statement or formula)** is a declarative sentence that is either true (denoted either T or 1) or false (denoted either F or 0) but not both.

(iii) The truth or falsehood of a logical proposition is called its **truth value.**

Notation: Variables are used to represent logical propositions. The most common variables used are p, q, and r.

Example 1.1.2.

$x + 2 = 2x$ when $x = -2$.

All cars are brown.

$2 \times 2 = 5$.

Here are some sentences that are not logical propositions (**paradox**).

Look out! **(Exclamatory)**

How far is it to the next town? **(Interrogative)**

$x + 2 = 2x$.

"Do you want to go to the movies?" **(Interrogative)**

"Clean up your room." **(Imperative)**

1.2. Truth Table

1.2.1. What is a Truth Table?

(i) A truth table is a tool that helps you analyze statements or arguments (defined later) in order to verify whether or not they are logical, or true.

(ii) A truth table of a logical proposition shows the condition under which the logical proposition is true and those under which it is false.

1.2.2. There are six basic operations called **connectives** that will utilize when creating a truth table. These operations are given below.

English Name	Math Name	Symbol
"and"	Conjunction	$\wedge$
"or"	Disjunction	$\vee$
"Exclusive"= "or but not both"	xor	$\underline{\vee}$
"if … then"	Implication	$\rightarrow$
"if and only if "	equivalence	$\leftrightarrow$
"not"	Negation	$\sim$

Definition 1.2.3. (Compound Statements)

If two or more logical propositions compound by connectives called compound proposition (statement). The truth value of a compound proposition depends only on the value of its components.

The rules for these connectives (operations) are as follows:

AND ($\wedge$) (conjunction): these statements are true only when both p and q are

AND $\wedge$ (Conjunction)		
p	q	$p \wedge q$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**