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- Let $S = \{a + b\sqrt{3} \mid a, b \in \mathbb{Z}\}$, then $(S, +, \cdot)$ is a subring of $(R, +, \cdot)$ where $\forall a, b, c, d \in \mathbb{Z}$

$$(a+b\sqrt{3}) - (c + d\sqrt{3})$$
$$= (a-c) + (b-d)\sqrt{3} \in S$$

$$(a+b\sqrt{3}) \cdot (c + d\sqrt{3})$$
$$= (ac+3bd) + (bc+ad)\sqrt{3} \in S$$

Def: Let $(R, +, \cdot)$ be any ring then the least positive integer n satisfy $na = 0$ $\forall a \in R$, is called characteristic of the ring.

if $na = 0$ $\forall a \in R \iff n = 0$ then we say that $(R, +, \cdot)$ has characteristic Zero.

Theorem: Let $(R, +, \cdot)$ be a ring with identity then $(R, +, \cdot)$ has characteristic $n > 0$ if and only if n the least positive integer for which $n1 = 0$