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$\Rightarrow$

proof: Let $n$ be the characteristic of $R$, then

$n1 = 0$ since ($na = 0$ $\forall a \in R$)

suppose $\exists m$ such that $0 < m < n$ and $m1 = 0$

then: $ma = m(1 \cdot a) = (m \cdot 1) \cdot a = 0 \cdot a = 0$ C!

$\Leftarrow$ Let $n1 = 0$, $n > 0$ $n$ is the least positive integer with this property

suppose $m$ is the characteristic of $R$,

$0 < m < n$ then $m \cdot 1 = 0$ C! $\begin{bmatrix} n \text{ is the least} \\ \text{positive} \ldots \end{bmatrix}$

thus $n$ is the characteristic of $R$.

Corollary: In Integral domain $(R, +, \cdot)$ all non-Zero elements have the same additive order which is the characteristic

proof: suppose $(R, +, \cdot)$ has positive characteristic $n$, then $\forall a \in R$ $(a \neq 0)$ $ma = 0$ for $m \leq n$

but $0 = ma = (m1) \cdot a$ therefore $m1 = 0$ (since $R$ is an Integral domain) thus $n \leq m$ (by last theorem)

hence $m = n$