Now,

if $(R,+,\cdot)$ has characteristic Zero then let $\forall a \in R$ $(a \neq 0)$, posses a finite additive order say $m \geq 0$. $0 = ma = (m1).a$

$\rightarrow m1 = 0 \rightarrow m = 0$ C!

thus each non-Zero element of R must be of Infinite order.

<u>Corollary</u>: the characteristic of an Integral domain $(R,+,\cdot)$ is either Zero or prime number.

<u>proof</u>: suppose $(R,+,\cdot)$ of positive characteristic n and n is not a prime then:

$n = n_1 \cdot n_2$ with $1 < n_i < n$ $(i = 1, 2)$

we have: $0 = n1 = (n_1 \cdot n_2)\, 1 = (n_1 \cdot n_2)\, 1^2$

$= (n_1 1)(n_2 1)$, since R is an Integral domain either $n_1 1 = 0$ or $n_2 \cdot 1 = 0$ C!

[n is the least positive integer with this property]

thus the characteristic must be prime.