Ideals and quotient rings

Def: Let $(R, +, \cdot)$ be a ring and I a non-empty subset of R then $(I, +, \cdot)$ is an ideal of $(R, +, \cdot)$ if and only if:

(1) $a - b \in I$, $\forall a, b \in I$

(2) If $r \in R$ and $a \in I$ then both $r \cdot a \in I$ and $a \cdot r \in I$.

Examples: (1) In any ring $(R, +, \cdot)$ the trivial subrings $(R, +, \cdot)$ and $(\{0\}, +, \cdot)$ are both ideals.

Remark: A ring which contains no ideals but the trivials called simple.

(2) The subring $(\{0, 3, 6, 9\}, +_{12})$ is an ideal of $(\mathbb{Z}_{12}, +_{12}, \cdot_{12})$ the ring of integers modulo 12.