Theorem: If $(I_i, +, \cdot)$ is an arbitrary indexed collection of ideals of the ring $(R, +, \cdot)$ then $(\bigcap I_i, +, \cdot)$ is also an ideal.

proof: since $0 \in I_i \ \forall i$ then $\bigcap I_i \neq \phi$

Let $a, b \in \bigcap I_i$ & $r \in R$ then $a, b \in I_i \ \forall i$

since $(I_i, +, \cdot)$ is an ideal $\forall i$ then

$a-b$, $r \cdot a$ and $a \cdot r \in I_i \ \forall i$ hence

$a-b$, $r \cdot a$ and $a \cdot r \in \bigcap I_i$ thus $(\bigcap I_i, +, \cdot)$ is an ideal.

– Let $(R, +, \cdot)$ be a ring and $\phi \neq S \subseteq R$ define

$$(S) = \bigcap \{ I \mid S \subseteq I \ ; \ (I, +, \cdot) \text{ is an ideal of } (R, +, \cdot) \}$$

– $(S) \neq \phi$ since $(R, +, \cdot)$ is one of its members.

– by last theorem the triple $((S), +, \cdot)$ is an ideal of $(R, +, \cdot)$ called the ideal generated by the set S.