- If $S = \{a\}$ then $(a) = \{r \cdot a \mid r \in R\}$ is called principle ideal $((a), +, \cdot)$ of $(R, +, \cdot)$.

theorem. If $(I, +, \cdot)$ is an ideal of the ring $(\mathbb{Z}, +, \cdot)$ then $I = (n)$ for some nonnegative integer n

proof: If $I = \{0\}$ then the zero ideal $(\{0\}, +, \cdot)$ is the principle ideal generated by 0,

Now, If $I \neq \{0\}$, then let $m \in I \Rightarrow -m \in I$

thus I contains positive integers, let n be the least positive integer in I, $(n) \subseteq I$ (since I is an ideal)

Let $k \in I$, $k = qn + r$ where $q, r \in \mathbb{Z}$ and $0 \leq r < n$

since $k, qn \in I$ therefore $k - qn = r \in I$

thus $r = 0$ (since n is the least positive integer in I)

then $k = qn$ and $I \subseteq (n)$ hence $I = (n)$.