Def: A principle ideal ring is a commutative ring with identity in which each ideal is principle

Example: Ring of integers $(\mathbb{Z},+,\cdot)$.

- Let $(R,+,\cdot)$ be a ring and $(I,+,\cdot)$ is an ideal of R, since R is commutative under addition $(I,+)$ normal subgroup of R, we define the cosets of I in R as follows

$$a+I=\{a+i \mid i\in I\} \quad a\in R$$

note that $a+I=b+I \iff a-b\in I$

let $R/I=\{a+I \mid a\in R\}$

the addition of cosets defined by:

$$(a+I)+(b+I)=(a+b)+I$$

$(R/I,+)$ is a commutative group