define multiplication by: $(a+I)\cdot(b+I)=(a\cdot b)+I$

it is well-defined where if

$$a+I=a'+I$$
$$b+I=b'+I$$

then $a-a'=i_1$ & $b-b'=i_2$ for some $i_1, i_2 \in I$

$$a\cdot b-a'\cdot b'=a\cdot(b-b')+(a-a')\cdot b'=a\cdot i_2+i_1\cdot b' \in I$$

thus $a\cdot b+I=a'\cdot b'+I$

theorem: If $(I,+,\cdot)$ is an ideal of the ring $(R,+,\cdot)$ then $(R/I,+,\cdot)$ is a ring called quotient ring of R by I.

proof: it is enough to know that the zero element is the coset $0+I=I$ & the inverse takes the form $-(a+I)=(-a)+I$