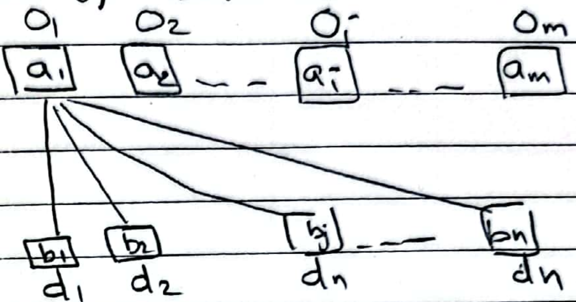
Transportation Problem (TP)

One of the application of LP is the TP

Description of the TP



m: no. of origins

n: no. of destinations

a_i: The amount of item available at origin i
عدد الوحدات المتوفرة في المخزن i

b_j: " " " " required at destination j
عدد الوحدات المطلوبة للمخزن الثانوي

C_{ij}: The cost of shipping one unit from origin to destination j
كلفة نقل الوحدة الواحدة من المخزن الرئيسي i الى المخزن الثانوي j

x_{ij}: The number of item shipped from origin i
عدد الوحدات المنقولة من المخزن الرئيسي i الى الثانوي j
destination j

Math. form of TP

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} = a_i \quad , \; i = 1, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , \; j = 1, \dots, n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0 \quad \forall \, i, j$$

It is clear that we have $m+n$ Constraints and n variables. The Constraints Can be written as:

x_{11}	x_{12}	x_{13}	...	x_{1n}	a_1
x_{21}	x_{22}	x_{23}	...	x_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_{m1}	x_{m2}	x_{m3}	...	x_{mn}	a_m
b_1	b_2	b_3	...	b_n	

Note: The rows represent the m Constraints
The Columns " " n "
Hence we have $m+n$ Constraints equations in the TP., one of these Constraints is redendent (extra), so the no. of Constraints is reduced to $m+n-1$.

The Solution of TP:

The TP is a special case of LPP, So it can be solved by Simplex method, but there is a special method, Can be used to solve the TP.

Algorithm for the sol. of TP

step(1) : Find initial basic feasible sol. (I.B.F.S.) by one of the methods

(1) North west Corner method
(2) Least Cost method
(3) Vogel's method

step(2) : Modified the I.B.F.S. by using Simplex mult-ipliers

step(3) : Check for optimality

① Searching for an I.B.F.S.

Choose any variable x_{pq} and make it as a basic variable with greatest value, can be found by

$$x_{pq} = \min\{a_p, b_q\}$$

1) If $a_p < b_q$, then the p th row is removed.
2) if $b_q < a_p$, then the q th Column " "
3) if $a_p = b_q$, then use (1) or (2).

Ex: Suppose we have 35 items in the 2 origins and to be shipped to 3 destinations as in the table

2	8 1	7 0	15
10 3	10 4	2	20
10	18	7	35

$\sum a_i = \sum b_j = 35$

no. of B.V.

$= m+n-1 = 4$

I.B.F.S. by least cost meth

$Z = \sum_i \sum_j C_{ij} x_{ij} = 78$

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