Lecture One

1. Introduction: The Real World vs. The Computer's World

To understand logic design, we first need to understand the fundamental language of computers.This begins with the distinction between **analogue** and **digital** signals.

- **Analogue:** The real world is analogue. This means values are continuous. Think of a dimmer switch for a light. You can smoothly turn the knob, and the light's brightness changes smoothly from off to fully on, passing through an infinite number of intermediate brightness levels. A mercury thermometer is another example—the height of the mercury rises continuously with temperature.
    - **Key characteristic: Infinite possibilities** between any two points.
- **Digital:** The computer world is digital. This means values are discrete. Think of a standard light switch. It has two distinct, separate states: ON or OFF. There's no in-between. A digital thermometer might show the temperature as "21.6°C". It's a specific, precise value, not a continuous column of liquid.
    - **Key characteristic: Finite, distinct states** (most commonly two).

2. Digital Number Systems

1. *Decimal*

This is the system we use in everyday life. It's believed to have originated from counting on our ten fingers.

- **Base:** 10
- **Digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- **Concept:** A number is represented as a sum of powers of 10.

2. *Binary*

This is the language of computers. As we saw with logic gates, digital circuits have two clear states: ON (1) and OFF (0). Binary maps perfectly onto this.

- **Base:** 2
- **Digits (Bits):** 0, 1
- **Concept:** A number is represented as a sum of powers of 2.

Each digit in a binary number is called a **bit** (binary digit). A group of 8 bits is a **byte**.

3. *Octal Number System*

This system has eight coefficients (0, 1, 2, 3, 4, 5, 6, 7) and the base is(8).

4. *Hexadecimal Number System*

The coefficients of this system are (0,1,2,3,4,5,6,7,8,9,A,B,C,E,F) and the base is (16) . Therefore if the **Base** or **Radix** of the system is R then the digits the base is (0,1,2,3,……….R-1) and for the base greater then 10, then symbols are used to represent the digits. In this case, letters, are used to represent the digits greater than (9).

3. *Number Base Conversion*

A. *Conversion from decimal system to other numbers system*

- ***Conversion from decimal system to (Binary)***

- The decimal number is converted to another system by dividing the decimal number by the base of another system and the process is continued until the integer quotient becomes 0. The remainders produced from this process represent the number in another system.

- **Example 1:** convert decimal 41 to binary?

العدد	الناتج	باقي القسمة
41	20	1
20	10	0
10	5	0
5	2	1
2	1	0
1	0	1

$(41)_{10}=(101001)_2$.

تؤخذ الارقام بالنظام الجديد من الاسفل الى الاعلى وتكتب من اليسار الى اليمين

- **Example 2:** convert decimal 25 to binary?

العدد	الناتج	باقي القسمة
25	12	1
12	6	0
6	3	0
3	1	1
1	0	1

$(25)_{10}=(11001)_2$

- **Example 3:** convert 53 decimals to binary?

العدد	الناتج	باقي القسمة
53	26	1
26	13	0
13	6	1
6	3	0
3	1	1
1	0	1

$(53)_{10}=(110101)_2$

- **<u>Converting Fractions from Decimal to Binary</u>**

1. Multiply the fraction by **2**
2. The digit before the decimal point in the result (either 0 or 1) becomes the next binary digit to the right.
3. Take the remaining fractional part and repeat the process.
4. Stop when the fractional part becomes **0**, or until you have reached your desired level of precision.

Example 1: Convert $(0.6875)_{10}$ to $(x)_2$?

Step	Operation	Result	Integer Part (Binary Digit)	Fractional Part to Carry Forward
1	0.6875 ×2	**1**.375	**1**	0.375
2	0.375 ×2	**0**.75	**0**	0.75
3	0.75 ×2	**1**.5	**1**	0.5
4	0.5 ×2	**1**.0	**1**	0.0

$(0.6875)_{10}$ to $(0.1011)_2$

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Example 2: Convert $(0.625)_{10}$ to $(x)_2$?

Step	Operation	Result	Integer Part (Binary Digit)	Fractional Part to Carry Forward
1	0.625 ×2	**1**.25	**1**	0.25
2	0.25 ×2	**0**.5	**0**	0.5
3	0.5 ×2	**1**.0	**1**	0.0

$(0.625)_{10}=(0.101)_2$

- ***Conversion from decimal system to (Octal)***

  Since this is a whole number, we will use the **repeated division by 8** methods (just like we did for binary, but dividing by 8 instead of 2).

Example 1: Convert $(37)_{10}$ to Octal number?

العدد	الناتج	باقي القسمة
37	4	5
4	0	4

$(37)_{10}=(45)_8$

Example 2: Convert $(145)_{10}$ to Octal number?

العدد	الناتج	باقي القسمة
145	18	1
18	2	2
2	0	2

$(145)_{10}=(221)_8$

- Conversion from decimal system to (Hexadecimal)

Example 1: Convert $(95)_{10}$ to Hexadecimal number?

العدد	الناتج	باقي القسمة
95	5	F
5	0	5

$(95)_{10}=(5F)_{16}$

Example 2: Convert $(119)_{10}$ to Hexadecimal number?

العدد	الناتج	باقي القسمة
119	7	7
7	0	7

$(119)_{10}=(77)_{16}$

B. Conversion from numbers system to Decimal system

- ***Conversion from Binary system to (Decimal)***

The general formula for converting any binary number to decimal is:

Formula:

$$\text{Decimal}=\sum_{i=0}^{n} b_i \times 2^i$$

Where:

- b_i is the binary digit (0 or 1) at position i
- 2^i is the place value
- Position i=0 is the **rightmost** digit (least significant bit)

Step-by-Step Method:

1. Write the binary number
2. Label each digit with its position number from right to left (starting at 0)
3. Multiply each digit by 2^position
4. Add all the results together

Example 1: Convert 1011_2 to decimal

text

Binary: 1 0 1 1

Position: 3 2 1 0

2^3 2^2 2^1 2^0

Value: 8 4 2 1

Calculation:

$(1\times2^3)+(0\times2^2)+(1\times2^1)+(1\times2^0)$

$=(1\times8)+(0\times4)+(1\times2)+(1\times1)$

$=8+0+2+1=11$

Result: $1011_2 = 11_{10}$

Example 2: Convert 11010_2 to decimal

text

Binary: 1 1 0 1 0

Position: 4 3 2 1 0

2^4 2^3 2^2 2^1 2^0

Value: 16 8 4 2 1

Calculation:

$(1\times16)+(1\times8)+(0\times4)+(1\times2)+(0\times1)$

$=16+8+0+2+0=26$

Result: $11010_2 = 26_{10}$

Example 3: Convert 100101_2 to decimal

text

Binary: 1 0 0 1 0 1

Position: 5 4 3 2 1 0

2^5 2^4 2^3 2^2 2^1 2^0

Value: 32 16 8 4 2 1

Calculation:

$(1\times32)+(0\times16)+(0\times8)+(1\times4)+(0\times2)+(1\times1)$

$=32+0+0+4+0+1=37$

$(100101)_2=(37)_{10}$

- ***Conversion from Octal system to (Decimal)***

Formula:

$$\text{Decimal}=\sum_{i=0}^{n} d_i \times 8^i$$

Where:

Where:

- di is the octal digit (0-7) at position i.
- 8i is the place value
- Position i=0 is the **rightmost** digit (least significant digit)

Step-by-Step Method:

1. Write the octal number
2. Label each digit with its position number from right to left (starting at 0)

3. Multiply each digit by 8^position
4. Add all the results together

Example 1: Convert 65_8 to decimal
Octal: 6 5
Position: 1 0
8^1 8^0
Value: 8 1
Calculation:
$(6\times8^1)+(5\times8^0)$
$=(6\times8)+(5\times1)$
$=48+5=53$
Result: 65_8 = **53_{10}**

Example 2: Convert 127_8 to decimal
Octal: 1 2 7
Position: 2 1 0
8^2 8^1 8^0
Value: 64 8 1
Calculation:
$(1\times8^2)+(2\times8^1)+(7\times8^0)$
$=(1\times64)+(2\times8)+(7\times1)$
$=64+16+7=87$
Result: 127_8 = **87_{10}**

- ***Conversion from hexadecimal system to (Decimal)***

Step-by-Step Method:
1. Write the hexadecimal number
2. Convert any letter digits (A-F) to their decimal values (10-15)
3. Label each digit with its position number from right to left (starting at 0)
4. Multiply each digit by 16^position
5. Add all the results together

Example 1: Convert $2F_{16}$ to decimal
Hex: 2 F
Decimal: 2 15
Position: 1 0
16^1 16^0
Value: 16 1
Calculation:
$(2\times16^1)+(15\times16^0)$

=(2×16)+(15×1)=(2×16)
=32+15=47
Result: $2F_{16}$ = $\mathbf{47_{10}}$

Example 2: Convert $A3C_{16}$ to decimal
Hex: A 3 C
Decimal: 10 3 12
Position: 2 1 0
16^2 16^1 16^0
Value: 256 16 1
Calculation:
(10×256)+(3×16)+(12×1)
=2560+48+12=2620
Result: $A3C_{16}$ = $\mathbf{2620_{10}}$

Example 3: Convert $1F4_{16}$ to decimal
Hex: 1 F 4
Decimal: 1 15 4
Position: 2 1 0
16^2 16^1 16^0
Value: 256 16 1
Calculation:
(1×256)+(15×16)+(4×1)
=256+240+4=500
Result: $1F4_{16}$ = $\mathbf{500_{10}}$

C. Conversion from **Binary system to other systems**

1. ***Conversion from Binary system to Hexadecimal***

Example 1: Convert $(1011010011110.11011101)_2$
000(1 0110 1001 1110. 1101 1101)
1 6 9 E . D D
Result $(1011010011110.11011101)_2$ =$(196E.DD)_{16}$

2. ***Conversion from Binary system to Octal***

Example 1 : Convert $(10101011.1101)_2$ to $(\ )_8$
010 101 011.110 100
2 5 3. 6 4
Result: $(10101011.1101)_2$ to $(253.64\ )_8$

والطريقة تكون بأخذ كل ثلاثة أرقام من اليمين الى اليسار بالنسبة للعدد قبل الفارزة ومن اليسار لليمين للعدد بعد الفارزة ونكمل بالرقم (0) ونعوض ما يقابل كل رقم بالنظام العشري.

D. Conversion from Octal system to Hexadecimal

To convert **Octal (base 8)** to **Hexadecimal (base 16)**, the easiest method is:
Octal → Binary → Hexadecimal

- Each **octal digit = 3 binary bits**
- Each **hex digit = 4 binary bits**

Example 1: Convert $(725)_8$ **to** Hexadecimal**.**

Step 1: Convert Octal to Binary

Each octal digit → 3 binary bits.

Octal	**Binary**
7	111
2	010
5	101

So:
$(725)_8=(111010101)_2$

Step 2: Group Binary into 4 bits (from right)

Add leading zero if needed:
0001 1101 0101

Step 3: Convert each group to Hexadecimal

Binary	**Hex**
0001	1
1101	D
0101	5

✅ **Final Result** $(725)_8=(1D5)_{16}$

E. Conversion from Hexadecimal to Octal

The common method is:
Hexadecimal → Binary → Octal

- Each **hex digit = 4 binary bits**
- Each **octal digit = 3 binary bits**

Example 1: Convert $\mathbf{(7B3)_{16}}$ to **Octal**.

Step 1: Convert Hexadecimal to Binary

Hex	**Binary**
7	0111
B	1011
3	0011

So:
$(7B3)_{16}=(011110110011)_2$

Step 2: Group Binary into 3 bits (from right)
011 110 110 011 011

Step 3: Convert each group to Octal

Binary	**Octal**
011	3
110	6
110	6
011	3

✅ **Final Result**
$(7B3)_{16}=(3663)_8$