Lecture Five
Canonical and Standard Forms in Boolean Algebra

1. Standard Forms

A standard form is any Boolean expression written as Sum of Products (SOP) or Product of Sums (POS).

- ✓ Sum of Products (SOP): OR of AND terms
  Example: F = AB + A'C + BC
- ✓ Product of Sums (POS): AND of OR terms
  Example: F = (A + B)(A' + C)(B + C)

2. Canonical Forms

A canonical form is a special type of standard form where every term contains all variables.

- ✓ Canonical SOP (Sum of Minterms): Each term includes all variables.
  Example: F(A,B,C) = A'B'C + A'BC + AB'C
- ✓ Canonical POS (Product of Maxterms): Each term includes all variables.
  Example: F(A,B,C) = (A + B + C')(A + B' + C)(A' + B + C)

Key Differences
Standard Form: May not include all variables, *usually simplified*.
Canonical Form: Must include all variables, *not simplified*.

Example Conversion
Given: F(A,B) = A + B
Canonical SOP: F = AB' + AB + A'B

1. Standard Forms

A Boolean function is defined using a truth table. The table contains rows where the output is either 1 (TRUE) or 0 (FALSE).

A. Sum of Products (SOP)

SOP is an OR (sum) of AND (product) terms. Each term (minterm) corresponds to a row where the output is 1.

Each minterm becomes 1 for exactly one combination of inputs.

By OR-ing all minterms, the function becomes 1 for all required rows.

Example:

F = A'B'C + A'BC

Key Point: SOP deals with rows where F = 1.

B. Product of Sums (POS)

POS is an AND (product) of OR (sum) terms. Each term (maxterm) corresponds to a row where the output is 0.

Each maxterm becomes 0 for exactly one combination of inputs.

By AND-ing all maxterms, the function becomes 0 for all required rows.

Example:

F = (A + B + C)(A + B' + C)

Key Point: POS deals with rows where F = 0.

C. Comparison

SOP: Works with 1's, uses OR of AND terms (minterms).

POS: Works with 0's, uses AND of OR terms (maxterms).

D. Conclusion

SOP collects all TRUE outputs, while POS eliminates FALSE outputs. This is why SOP is based on 1's and POS is based on 0's.

E. Examples

I. From the given truth table express F as a SOP and product of POS.

No.	X	Y	Z	F	SOP	POS
1	0	0	0	0		X+Y+Z
2	0	0	1	0		X+Y+Z'
3	0	1	0	1	X'YZ'	
4	0	1	1	1	X'YZ	
5	1	0	0	0		X'+Y+Z
6	1	0	1	0		X'+Y+Z'
7	1	1	0	1	XYZ'	
8	1	1	1	0		X'+Y'+Z'

SOP

F=X’YZ’+X’YZ+XYZ’

$$F(m3, m4, m7) = \sum(3,4,7)$$

POS

F=(X+Y+Z)(X+Y+Z’)(X’+Y+Z)(X’+Y+Z’)(X’+Y’+Z’)

$$F(m1, m2, m5, m6, m8) = \prod(1,2,5,6,8)$$

II. From the given truth table express F as a SOP and POS.

No.	A	B	C	F	SOP	POS
1	0	0	0	0		A+B+C
2	0	0	1	1	A’B’C	
3	0	1	0	0		A+B’+C
4	0	1	1	0		A+B’+C’
5	1	0	0	1	AB’C’	
6	1	0	1	1	AB’C	
7	1	1	0	1	ABC’	
8	1	1	1	1	ABC	

SOP

From truth table, rows where F=1:

F = A'B'C + A'BC + AB'C' + AB'C + ABC' + ABC

POS

From truth table, rows where F=0:

F = (A + B + C) (A + B ‘+ C)(A + B' + C')

2. Canonical Forms

I. Express the Boolean Function F=A+B'C in a **sum of minterms**.
Solution:
the function F has three variables A, B and C, it is in SOP standard form the first product term (A) missing two variable (B,C); therefore
F=A(B+B’)+B’C
F=AB+AB’+B’C

F=AB(C+C’)+AB’(C+C’)+B’C
F=ABC+ABC’+AB’C+AB’C’+B’C
F=ABC+ABC’+AB’C+AB’C’+B’C(A+A’)
F=ABC+ABC’+AB’C+AB’C’+AB’C+A’B’C
F=ABC+ABC’+AB’C’+A’B’C
= m1 + m4 + m5 + m6 + m7
F(A,B,C) ∑(1, 4, 5, 6, 7)

II. Express the Boolean Function F=xy + x'z in a product of maxterms.

Solution:
First: convert the function into OR terms (POS) by using distributive law:
F= (xy + x')(xy + z) = (x' + x)(x' + y)(z + x)(z + y)
F= (x' + y)(z + x)(z + y)
The function has three variables x, y and z. each OR term is missing one variable; therefore:
(x'+y) = (x'+y) +zz' = (x'+y+z) (x'+y+z')
(z + x) = (z + x) + yy' = (z + x +y) (z + x +y') = (x+y+z) (x+y'+z)
(z + y) = (z + y) + xx' = (z + y + x) (z + y + x') = (x+y+z) (x'+y+z)
Combining all terms and removing all those that appear more than once, we finally obtain:
F = (x+y+z) (x+y'+z) (x'+y+z) (x'+y+z') = M0, M2, M4, M5
F(x, y, z) = ∏ (0, 2, 4, 5)

III. Express the Function F in SOP then simplify it.

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Sum of minterms

$$F(X,Y,Z) = \overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,\overline{Y}\,Z + X\,Y\,\overline{Z} + X\,Y\,Z$$

$$= \overline{X}\,Y(\overline{Z} + Z) + X\,\overline{Y}\,Z + X\,Y(\overline{Z} + Z)$$

$$= \overline{X}\,Y + X\,\overline{Y}\,Z + XY = Y(\overline{X} + X) + X\,\overline{Y}\,Z = Y + X\,\overline{Y}\,Z$$

$$= X\,\overline{Y}\,Z + Y = (XZ + Y)(\overline{Y} + Y) = XZ + Y$$

H.W.

1. Obtain the canonical product of the sum form of the following function.

   $F(A, B, C) = A + B'C$

2. Obtain the canonical product of the sum form of the following function.

   $F(A, B, C) = (A + B')(B + C)(A + C')$

3. Obtain the canonical sum of product form of the following function.

   $F(A, B) = A + B$

4. Prove the following using Boolean theorems:

   (a) $(A + C)(A + D)(B + C)(B + D) = AB + CD$

   (b) $(X + Y) \oplus (X + Z) = X'(Y \oplus Z)$.

5. Find the Boolean expression for F, when F is 1 only if A is 1 and B is 1, or if A is 0 and B is 0.

6. Convert $F = ABCD + A'BC + B'C'$ into a sum of minterms by algebraic method.

7. Convert $F = AB + B'CD$ into a product of maxterms by algebraic method.