حلول اسئلة امتحان المد الثاني

1- If $0=\emptyset,\ 1=\{\emptyset\}$ compute 6

Solution:

6={0,1,2,3,4,5}={Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}},{Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}}
},{Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}}},{Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}}}}}

2- find $(1+4^+)^+$

Solution: $(1+4^+)^+=(1+4)^{++}=(5)^{++}=6^+=7$

3- Find 1+5 in $\mathbb{Z}$

Solution: $1+5=[1,0]+[5,0]=[1+5,0+0]=[6,0]=6$

4- Show that $2<4$ in $\mathbb{Z}$

Solution: $2=[2,0]<[4,0]=4$, because $2+0<4+0$ in $\mathbb{N}$ there exists 2 such that $2+2=4$

5-Show that $(2\cdot 3)^+<(3^+\cdot 5)$

Solution: $(2\cdot 3)^+=(2\cdot 2^+)^+=(2+2\cdot 2)^+=(2+4)^+=6^+=7$

And $(3^+\cdot 5)=(5\cdot 3^+)$ ($\cdot$ is commutative)

$=(5+5\cdot 3)=5+15=20$

That is $7<20$ because $\exists\ 13\in\mathbb{N}\ s.t\ 7+13=20$

Remark: $(a\cdot b)^+\neq(a\cdot b^+)$

Example: $(3\cdot 5)^+=(3\cdot 4^+)^+=(3+3\cdot 4)^+=(3+12)^+=15^+=16$

And $3\cdot 5^+=3+3\cdot 5=3+15=18.$

6-prove that if $n\neq 0, then\ 1<n^+, 0+n=n$

Solution: By definition $x<y$ iff there exist $k(\neq 0)\in\mathbb{N}$ such that $x+k=y$.

And snice $\neq 0$, $1+n=n^+(by\ theorem)$ then $1<n^+$ as $k=n$. And since + is commutative then $0+n=n+0=n$

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7-Prove that: If $a = [l, 0]$ $l \neq 0, b = [r, s] \in \mathbb{Z}$ such that $b \odot a = a$, then $b = [1,0]$

Solution: Since $b \odot a = a$, then $l \neq 0$ and

$$[r, s] \odot [l, 0] = [l, 0]$$

$\Longrightarrow \quad [r \cdot l + s \cdot 0, s \cdot l + r \cdot 0] = [l, 0]$

$\Longrightarrow \quad [r \cdot l, s \cdot l] = [l, 0] \quad \Longrightarrow (r \cdot l, s \cdot l) R^* (l, 0)$

$\Longrightarrow r \cdot l + 0 = s \cdot l + l \overset{(\text{dist. Law for } \cdot)}{\Longrightarrow} r \cdot l = (s + 1) l \overset{(\text{Cancel, Law for } \cdot)}{\Longrightarrow} r = s + 1$

$\Longrightarrow r + 0 = s + 1 \Longrightarrow (r, s) R^* (1,0) \Longrightarrow b = [r, s] = [1,0]$

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