linear Algebra 2

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Lecture2

Eigenvectors and Eigenvalues

$= (1-\lambda)(-2-\lambda+\lambda^2+1) + (-3-3\lambda+2) + (12-16+8\lambda)$

$= -2-\lambda+\lambda^2+1+2\lambda+\lambda^2-\lambda^3-\lambda-1-3\lambda-4+8\lambda$

$= -\lambda^3 + 2\lambda^2 + 5\lambda - 6$

Therefore the characteristic polynomial of the matrix B is $-\lambda^3+2\lambda^2+5\lambda-6$ and the characteristic equation of the matrix B is $-\lambda^3+2\lambda^2+5\lambda-6=0$.

$-\lambda^3+2\lambda^2+5\lambda-6=0$

$\Rightarrow (-\lambda+1)(-\lambda-2)(-\lambda+3)=0$

$\Rightarrow -\lambda+1=0$ or $-\lambda-2=0$ or $-\lambda+3=0$

$\Rightarrow \lambda=1$ or $\lambda=-2$ or $\lambda=3$

Thus $\lambda=1$ and $\lambda=-2$ and $\lambda=3$ are the eigenvalues of the matrix B.

Remarks:

1) The characteristic equation of a 2×2 matrix D is $a_2\lambda^2+a_1\lambda+a_0=0$ where $a_2=1$, $a_1=-d_{11}-d_{22}$, and $a_0=|D|$.

2) The characteristic equation of a 3×3 matrix E is $a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0=0$ where $a_3=-1$, $a_2=e_{11}+e_{22}+e_{33}$,

$a_1 = -\begin{vmatrix} e_{22} & e_{23} \\ e_{32} & e_{33} \end{vmatrix} - \begin{vmatrix} e_{11} & e_{13} \\ e_{31} & e_{33} \end{vmatrix} - \begin{vmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{vmatrix}$, and

$a_0 = |E|$.

9. **Example**: Given that the characteristic equation of the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$ is $\lambda^2 - 8\lambda + 7 = 0$. Find

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i) the eigenvalues of the matrix A.

ii) the eigenvectors of the matrix A.

Solution:

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i) $\lambda^2 - 8\lambda + 7 = 0$

$\Rightarrow (\lambda - 7)(\lambda - 1) = 0$

$\Rightarrow \lambda - 7 = 0$ or $\lambda - 1 = 0$

$\Rightarrow \lambda = 7$ or $\lambda = 1$

$\Rightarrow \lambda = 7$ and $\lambda = 1$ are the eigenvalues of the matrix A.

ii) 1- We find the eigenvectors corresponding to the eigenvalue $\lambda = 7$ as follows:

$AU = \lambda U$

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 7 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 7 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x_1 + 5x_2 \\ x_1 + 6x_2 \end{pmatrix} - \begin{pmatrix} 7x_1 \\ 7x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x_1 + 5x_2 - 7x_1 \\ x_1 + 6x_2 - 7x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -5x_1 + 5x_2 \\ x_1 - x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{matrix} -5x_1 + 5x_2 = 0 & \cdots (1) \\ x_1 - x_2 = 0 & \cdots (2) \end{matrix}$$

10. From (1) and (2), we get that $x_1 = x_2$. So the eigenvectors corresponding to the eigenvalue $\lambda = 7$ are $v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, for all $x_2 \in \mathbb{C}$.

$\begin{pmatrix} k x_2 \\ k x_2 \end{pmatrix}$, $\forall\, k \in \mathbb{C}$ constant

2- We find the eigenvectors corresponding to the eigenvalue $\lambda = 1$ as follows:

$AV = \lambda V$

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x_1 + 5x_2 \\ x_1 + 6x_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x_1 + 5x_2 - x_1 \\ x_1 + 6x_2 - x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1 + 5x_2 \\ x_1 + 5x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} x_1 + 5x_2 &= 0 \quad \text{--- (1)} \\ x_1 + 5x_2 &= 0 \quad \text{--- (2)} \end{aligned}$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} 9+7i \\ 9+7i \end{pmatrix} = \begin{pmatrix} 2(9+7i) + 5(9+7i) \\ 1(9+7i) + 6(9+7i) \end{pmatrix} = \begin{pmatrix} 7(9+7i) \\ 7(9+7i) \end{pmatrix}$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} -5(9+7i) \\ 9+7i \end{pmatrix} = \begin{pmatrix} -10(9+7i) + 5(9+7i) \\ -5(9+7i) + 6(9+7i) \end{pmatrix} = \begin{pmatrix} -5(9+7i) \\ 9+7i \end{pmatrix} = 1 \begin{pmatrix} -5(9+7i) \\ 9+7i \end{pmatrix}$$

From (1) and (2), we get $x_1 = -5x_2$. So the eigenvectors corresponding to the eigenvalue $\lambda = 1$ are $v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -5x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -5 \\ 1 \end{pmatrix}$, for all $x_2 \in \mathbb{C}$.

Properties of Eigenvalues and Eigenvectors

1) The sum of the eigenvalues of a matrix is

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11. equal to its trace, which is the sum of the elements on its main diagonal.

2) A matrix and its transpose has the same eigenvalues.

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$A^T = ($

3) The eigenvalues of an upper or lower triangular matrix are the elements on its main diagonal.

المعقوفه و trans نفس eigenvalues

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4) The product of the eigenvalues of a matrix equals the determinant of the matrix.

5) If v is an eigenvector of a matrix A corresponding to the eigenvalue λ, then kv is also an eigenvector of the matrix A corresponding to the same eigenvalue λ, for any nonzero constant k.

§2: Similar Matrices

Definition: Let A and B be two $n \times n$ matrices. We say that B is similar to A if there is an invertible matrix P such that $B = P^{-1}AP$.

شبيه

Remark: If a matrix A is similar to a matrix B then the matrix B is similar to the matrix A → B is similar to A

$B = P^{-1}AP$

تضرب ب P

$PB = PP^{-1}AP$

$PB = AP$

$PBP^{-1} = APP^{-1}$

$A = (P^{-1})^{-1}BP$

A is similar B