linear Algebra 2

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Lecture3

Similar matrices

مجموع يساوي القطر الرئيسي

11. equal to its trace, which is the sum of the elements on its main diagonal.

2) A matrix and its transpose has the same eigenvalues.

$A^T = ($

3) The eigenvalues of an upper or lower triangular matrix are the elements on its main diagonal.

المعقوفة و trans نفس eigenvalues

eigenvalues حاصل ضرب eigenvalues

4) The product of the eigenvalues of a matrix equals the determinant of the matrix.

5) If v is an eigenvector of a matrix A corresponding to the eigenvalue λ, then kv is also an eigenvector of the matrix A corresponding to the same eigenvalue λ, for any nonzero constant k.

§2: Similar Matrices

Definition: Let A and B be two $n \times n$ matrices. We say that B is similar to A if there is an invertible matrix P such that $B = P^{-1}AP$.

Remark: If a matrix A is similar to a matrix B then the matrix B is similar to the matrix A → B is similar to A

$B = P^{-1}AP$

تقريب P ... $PB = PP^{-1}AP$

$PB = AP$

$PBP^{-1} = APP^{-1}$

$A = (P^{-1})^{-1}BP$

A is similar B

$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$

12. <u>Example 1</u>: Give two different matrices A and B similar to the matrix $D = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.

<u>Solution</u>: Let $P_1 = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, then $P_1^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

Hence A can be

$$A = P_1^{-1} D P_1 = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 7 & 12 \\ 13 & 22 \end{pmatrix} = \begin{pmatrix} -25 & -42 \\ 19 & 32 \end{pmatrix}$$

Thus $A = \begin{pmatrix} -25 & -42 \\ 19 & 32 \end{pmatrix}$ is similar to $D = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.

Now let $P_2 = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$, then $P_2^{-1} = \begin{pmatrix} 3 & -7 \\ -2 & 5 \end{pmatrix}$.

Hence B can be

$$B = P_2^{-1} D P_2 = \begin{pmatrix} 3 & -7 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -7 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 16 & 23 \\ 30 & 43 \end{pmatrix} = \begin{pmatrix} -162 & -232 \\ 118 & 169 \end{pmatrix}$$

Thus $B = \begin{pmatrix} -162 & -232 \\ 118 & 169 \end{pmatrix}$ is similar to $D = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.

<u>Example 2</u>: Give two different matrices A and B similar to the matrix $C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 8 \end{pmatrix}$.

<u>Solution</u>: Let $P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, then

$$P_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

13. Hence A can be

$$A = P_1^{-1} C P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 8 \\ 4 & 11 & 17 \\ 5 & 15 & 23 \end{pmatrix} = \begin{pmatrix} 1 & 5 & 8 \\ 3 & 7 & 11 \\ 1 & 4 & 6 \end{pmatrix}.$$

Thus $A = \begin{pmatrix} 1 & 5 & 8 \\ 3 & 7 & 11 \\ 1 & 4 & 6 \end{pmatrix}$ is similar to $C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 8 \end{pmatrix}$.

Now let $P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ then $P_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

Hence B can be

$$B = P_2^{-1} C P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 5 \\ 4 & 5 & 11 \\ 5 & 7 & 15 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ -1 & -2 & -4 \\ 5 & 7 & 15 \end{pmatrix}$$

Thus $B = \begin{pmatrix} 1 & 2 & 5 \\ -1 & -2 & -4 \\ 5 & 7 & 15 \end{pmatrix}$ is similar to

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 8 \end{pmatrix}.$$

14\. Properties of Similarity: مهم اعتيادي — قانون

1) Any matrix is similar to itself (i.e. any matrix A is similar to the matrix A).
2) If a matrix A is similar to a matrix B, then the matrix B is similar to the matrix A.
3) If a matrix A is similar to a matrix B and the matrix B is similar to a matrix C, then the matrix A is similar to the matrix C.

§3: Diagonalization

Definition: An $n\times n$ matrix A is called diagonalizable if A is similar to a diagonal matrix B.

Theorem 2: If A is an $n\times n$ matrix, then A is diagonalizable iff A has n linealy independent eigenvectors.

Example: Show that the matrix $A=\begin{pmatrix}12 & 10\\ 1 & 3\end{pmatrix}$ is diagonalizable.

Solution: $Au=\lambda u \Rightarrow Au=\lambda I u \Rightarrow (A-\lambda I)u=0$

$\Rightarrow \left(\begin{pmatrix}12 & 10\\ 1 & 3\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}\right)\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$.

$\lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$