linear Algebra 2

Dr.Jawad Kadhim Khalaf Al-Delfi

Lecture5

Linear programming

البرمجة الخطية
LINEAR PROGRAMMING

Linear programming is a particular class of programming problems which satisfies the following conditions:

1. The decision variables which we are dealing with in the problem are non-negative (i.e. positive or zero).
2. For selecting the best values of the decision variables, we can describe these variables by a linear function. This function is called the objective function.
3. The operating rules governing the process can be expressed as a set of linear equations or linear inequalities. This set is called the constraint set.

The term linear programming used because of the second and third conditions.

Notice that linear programming are widely used for solving military, economic, industrial and social problems.

The basic steps in constructing a linear programming model are the following three steps:

i) Identify the unknown variables (decision variables) and represent them in terms of algebraic symbols.

ii) Identify all the restrictions or constraints in the problem and express them as a linear equations or inequalities which are linear function of the unknown variables.

iii) Identify the objective or criterion and represent it as a linear function of the decision variables, which is to be maximized or minimized.

Example 1:

A shirt company has two kinds of workers, 1 and 2. It is required that at least 540 shirts to be produced from the company per 8-hours day. Grade 1 workers can

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produce 2.5 shirts per hour with an accuracy of 95%. Grade 2 workers can produce 1.5 shirts per hour with accuracy of 98%.

The wage of a grade 1 worker is ID 4,000 per hour, while that of a grade 2 worker is ID 3,000 per hour. Each unperfect shirt produced from the company cost the company ID 2.000

The company has available for the jobs 24 grade 1 workers and 30 grade 2 workers

The company wants to determine the optimal assignment of workers which will minimize the total cost of the production.

Formulation:

Step (i): Identify the decision variables.

The unknown variables are the number of grade 1 and grade 2 workers, which we represent them by the following algebraic symbols:

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x_1 = the number of grade 1 workers

x_2 = the number of grade 2 workers.

Step (ii): Identify the constraints. In this problem the constraints are the limited of each grade of workers, and the quantity of production. Therefore we have the following constraints for the number of workers

$x_1 \leq 24$

$x_2 \leq 30$

and we have the following constraint for the quantity of the production

$8(2.5)x_1 + 8(1.5)x_2 \geq 540$ or

$20x_1 + 12x_2 \geq 540$

Step (iii): Identify the objective function.

In order to develop the objective function, we note that the company have two type of costs during the production; wages paid to the workers, and the cost of their errors.

The hourly costs of each grade 1 worker is

ID 4,000 + ID 2.000 (2.5)(0.05) = ID 4,250

Minimize: $Z = 8(4.25)x_1 + 8(\quad)$

$= 4000x_1 + 2445x_2$

each raid of type B results in the destruction of 10 bases and requires 3 planes and 6 rocket bombs per each destroyed base. If the available number of planes is 14 and the available rocket bombs is 210. Formulate a linear programming model in order to maximize the total bases to be destroyed. Notice that all the air-raids are to be done at the same time, i.e. each plane is involved only once in the air-raids.

Formulation:

Step (i): Identify the decision variables. The unknown activities to be determined are the number of air-raids of each type, and we will represent them by algebraic symbols.

x_1 = The number of air-raids of type A

x_2 = The number of air-raids of type B.

Step (ii): Identify the constraints.

Similarly for each grade 2 worker

$ID\,3{,}000 + ID\,2{,}000(1.5)(0.02) = ID\,3.060$

Thus the objective function is to minimize the daily cost of workers

$$Z = 8(4{,}250x_1 + 3{,}060x_2)$$
$$= 34{,}000x_1 + 24{,}480x_2$$

The complete formulation of the linear programming problem thus becomes

Minimize: $Z = 34{,}000x_1 + 24{,}480x_2$

Subject to: $x_1 \leq 24$

$x_2 \leq 30$

$20x_1 + 12x_2 \geq 540$

$5x_1 + 3x_2 \geq 135$

$x_1 \geq 0,\ x_2 \geq 0$.

Example 2:

Military bases are under two types of air-raids A and B. Each raid of type A results in the destruction of 6 bases and requires 5 planes and 5 rocket bombs per each destroyed base, while

7. In this problem the constraints are the limited availability of the two resources, planes and rocket bombs.

Type A requires 5 planes for each air-raid, and its number of raids is x_1. Hence the requirement of planes for type A alone will be $5x_1$.

Similarly type B will require $3x_2$.

Thus the total requirement of planes will be $5x_1+3x_2$, which should not exceed the available 14 planes. So the plane constraint becomes

$$5x_1+3x_2 \leq 14$$

Similarly the rocket bombs requirement will be $6.x_1 = 30x_1$ for type A, and $6.10.x_2 = 60x_2$ for type B. Thus the rocket bombs constraint becomes

$$30x_1+60x_2 \leq 210$$

Also we restrict the variables x_1 and x_2 to have only non-negative values.

Step (iii): Identify the objective.

8. The objective is to maximize the total number of bases to be destroyed. This becomes

$$Z = 6x_1 + 10x_2$$

Thus the linear programming model for our problem becomes:

Find the integers x_1 and x_2 which will

Maximize: $Z = 6x_1 + 10x_2$

Subject to: $5x_1 + 3x_2 \leq 14$

$30x_1 + 60x_2 \leq 210$

$x_1 \geq 0, x_2 \geq 0$

In the above two examples we showed you how practical problems can be formulated mathematically as linear programming problems. The next step after formulation is to solve the problem mathematically to obtain the best possible solution