linear Algebra 2

Dr.Jawad Kadhim Khalaf Al-Delfi

Lecture6

Graphical method

Graphical solution of linear programs in two variables:

We discuss now how to solve linear programming problems involving only two variables by using the graphical procedure. This procedure is presented to illustrate some of the basic concepts used in solving large linear programming problems. Before discussing the procedure let us have some definitions.

Definition: A given values to the dicision variables which satisfies all the constraints is called a feasible solution. حل مقبول

Definition: The set of all feasible solutions is called the feasible region. منطقة الحلول المقبولة

Definition: A best feasible solution is called an optimal solution. حل امثل

Definition: The value of the objective function corresponding to an optimal solution is called the optimal value of the linear program. القيمة المثلى

Definition: A linear program which has only one optimal solution is called a linear program with unique optimal solution.

Definition: A linear program which has more than one feasible solution such that their objective function values are equal to the optimal value, the linear program is said to have a multiple optimal solutions.

Definition: When there exists no finite optimal solution, the linear program is said to have an unbounded solution.

Now to show how we use the graphical procedure let us discuss the following example.

The solution of example 1 :

The formulation of example 1 is

Minimize : $Z = 340x_1 + 24.48x_2$

Subject to : $x_1 \leq 24$

$$x_2 \leq 30$$

$$5x_1 + 3x_2 \geq 135$$

$$x_1 \geq 0 \ , \ x_2 \geq 0$$

We want to determine the values of the variables x_1 and x_2 which satisfy all the restrictions and give the least value for the objective function.

To represent the feasible region in a graph, every constraint is plotted, and all the values of x_1 and x_2 that will satisfy these constraints are identified. The constraints $x_1 \geq 0$ and $x_2 \geq 0$ implies that all feasible solutions lies in the first quadrant.

The constraint $5x_1 + 3x_2 \geq 135$ means that any feasible solution (x_1, x_2) to the problem should be on one side of the straight line $5x_1 + 3x_2 = 135$. The proper side is found

by testing whether the origin satisfies the constraint or not. The line $5x_1 + 3x_2 = 135$ is plotted by taking two convenient points (e.g. $x_1 = 0$, $x_2 = 45$ and $x_1 = 27$, $x_2 = 0$). The proper side is indicated by an arrow directed above the line since the origin does not satisfy the constraint. Similarly the constraint $x_1 \leq 24$ and $x_2 \leq 30$ are plotted. The feasible region is given by the shaded region ABC.

Obviously there is infinite number of feasible points in this region. Our objective is to identify the feasible point with the lowest value of Z.

Represent the objective function $Z = 34x_1 + 24.48x_2$ by a straight line if the value of Z is fixed. Changing the value of Z translates the entire line to another straight line parallel to itself. We can choose the value 1200 as an initial value of Z to plote the straight line. By moving this line closer to the origin the value of Z is olution of example 2 :

ormulation of example 2

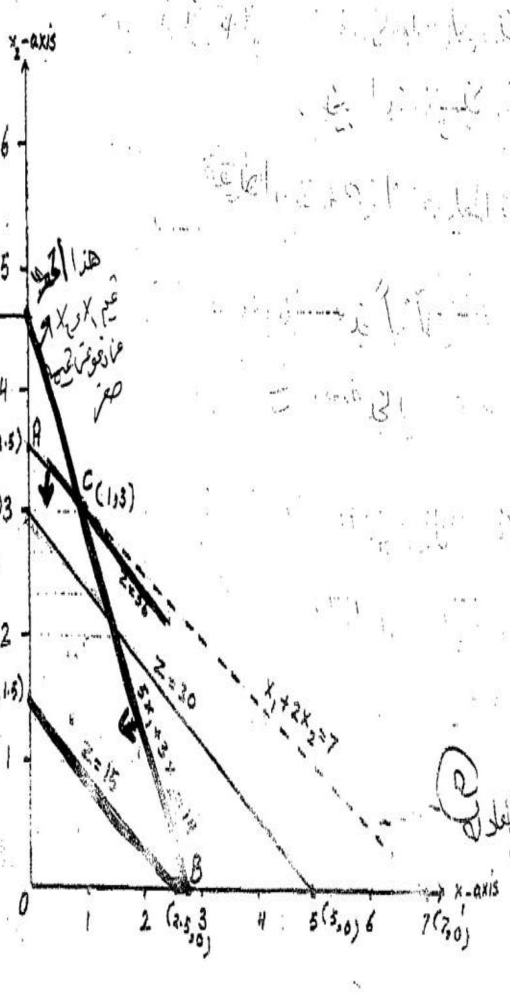
nize : $Z = 6x_1 + 10x_2$

ct to : ① ... $5x_1 + 3x_2 \geq 14$

② ... $x_1 + 2x_2 \leq 7$

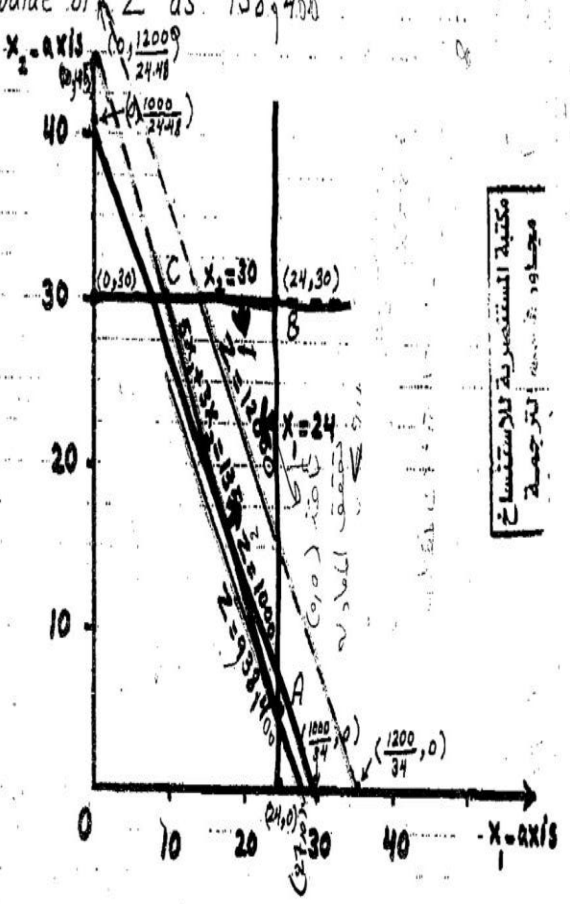
$x_1 \geq 0 \, , \, x_2 \geq 0$

easible region is shown in the following



13.

further decreased. The optimal solution is when the straight line Z is the nearest to the origin and also cutting the feasible region and this is occure at the corner point A given by $x_1 = 24$, $x_2 = 5$. This is the best feasible solution giving the lowest value of Z as 938,400



Hence $x_1 = 24$ and $x_2 = 5$ is an optimal solution and $Z = 938,400$ is the optimal value for the linear program.

The objective function lines are drawn for $Z = 15, 30$ and 36. The optimal value for the linear program is 36, and the corresponding objective function line $6x_1 + 10x_2 = 36$ pass through the point C of the feasible region. Thus the feasible solution $x_1 = 1$ and $x_2 = 3$ is the optimal solution.

Definition: A linear programming problem with m constraints and n variables which can be represented as follows:

Maximize (or Minimize): $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

القيود | Subject to:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$
$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$
$$b_1 \geq 0, b_2 \geq 0, \ldots, b_m \geq 0$$

is called a linear program in standard form.

i.e. The linear program is in standard form if it is satisfy the following:

i) The objective function is of the maximization or minimization type.

ii) All the constraints are expressed as equations. القيود جميعها معادلات

iii) All the variables are non-negative متغيرات غير سالبة

iv) The constant which is on the right hand side of each constraint equation is non-negative. جميع النواتج في المعادلات

الارقام الموجودة على جهة اليمين من كل معادلة غير سالبة ≥ 0

Converting a linear programming problem to a problem in standard form:

Since not all the linear programming problems come in the standard form and usually the constraints in the linear program expressed as inequalities rather than equations and some of its decision variables to be not non-negative. Hence to convert the linear program to the standard form we need to go through the following steps:

1) Handling inequality constraints: